Maxwell's Legacy

James C. Rautio

Working with RF has been a passion of mine ever since I learned how to grab a soldering iron by the right end. Numerical electromagnetics (EM) has held my attention for the last two decades. Gradually, as I started playing with Maxwell's equations back in the "good old days," I began wondering about this Maxwell fellow (Figure 1). Historians acknowledge him to be the most important physicist of the 19th century, the equal of Einstein and Newton. Any bookstore or library has biographies of both Newton and Einstein. . . so where are the Maxwell biographies?

Indeed, biographies of Maxwell are difficult to find. Over the last decade or so, I have made an effort to find some and, in my "spare time" between running a company, programming, writing papers, and doing research, I have made a point to learn a bit about the man who was the founder of our field. While I am not a historian by any measure, I can share with you what I have learned from my viewpoint as a microwave engineer. One way I am achieving this is as a volunteer MTT-S Distinguished Microwave Lecturer speaking on "The Life of James Clerk Maxwell" [1]. In this article, I discuss the events leading up to Maxwell's most important legacy in more detail than time permits in my lecture. Interestingly, his most important legacy is not the displacement current. It's not even Maxwell's equations.

Perhaps the best source of information on Maxwell's life is an 1882 biography by Lewis Campbell and William Garnett [2]. Taken from the biography, Figure 2 is a sketch made from a watercolor that was painted from life. This shows Maxwell's interest in wave phenomena (the violin) even as a child. Normally, this book is available only in the rare book rooms of large libraries. However, back when I was first becoming interested in Maxwell, I obtained a copy and scanned and converted it to text, a fairly lengthy process. Today, you can download a free PDF of the entire biography, including unique portions of the second edition, from the Web [3].

---

*James C. Rautio (rautio@sonnetsoftware.com) is with Sonnet Software, Inc.
100 Elwood Davis Road, North Syracuse, NY 13212 USA.*

1527-3342/05/$20.00©2005 IEEE

Campbell foreshadows Maxwell's future work in this passage about Maxwell as a child:

> But his most obvious interests were naturally out of doors . . . to leap ditches, to climb trees "of sorts," to see them felled and "have grand game at getting upon them when falling," to take wasps' nests on hot days in July, to blow soap bubbles and marvel at their changing hues . . .

I'm sure jumping on falling trees and playing with wasp nests was exciting for the young boy, but it was his interest in soap bubbles that spoke to the future.

The nature of light has been a topic of speculation since before Aristotle. In the century before Maxwell, the theories of Newton and Huygens (Figure 3) competed for the mind of the natural philosopher, as physicists were then known. Newton believed light was tiny corpuscles, with color coming from vibrations caused by the corpuscles speeding thorough the luminiferous æther (modern spelling, ether). Huygens proposed that light was entirely a wave phenomenon, the waves propagating through the æther.

Thomas Young showed that the colors on a soap bubble could be the result of wave interference, supporting Huygen's wave theory. However, there was some difficulty with this hypothesis, which Maxwell saw as a 15-year old. As reported by Campbell:

> In the spring of 1847 . . . his uncle, Mr. John Cay, . . . took James and myself . . . to see Mr. Nicol, a friend of Sir David Brewster, and the inventor of the polarising prism. Even before this James had been absorbed in "polarised light," working with Iceland spar, . . . but this visit added a new and important stimulus to his interest in these phenomena, and the speculations to which they give rise.

Iceland Spar, a clear crystalline form of calcite, is doubly refractive. There is one index of refraction for one polarization and a second index for the orthogonal polarization. If you draw a line on a piece of paper and place the crystal over it, you see two lines. Given an unpolarized ray of light incident on the crystal, it is split into an "ordinary ray" and an "extraordinary ray." Nicol found a way to use two prisms of Iceland Spar glued together to separate the two rays.

Nicol was so impressed with young Maxwell that he gave Maxwell two sets of polarizing prisms. Returning home, Maxwell melted glass and poured it into a mold. He then cooled the glass quickly, leaving significant stress in the glass. He placed one prism in back of the glass (the "polarizer") and a second prism in front of the glass (the "analyzer"). He built a camera lucida and, by his own hand, made watercolors of the stress patterns in the glass, as shown in Figure 4. Then he solved Stoke's equations and compared the measured versus calculated results in a published paper.

This incident is significant for two reasons. First, Maxwell was actually working with fields. Stress and strain are purely mechanical and very physical fields; there is nothing abstract about them, but they are fields (a field being a vector defined over a space).

The second significant aspect is a glaring problem with the wave theory of light that this experiment illustrated. Sound is a longitudinal wave. The air vibrates back and forth along a line from the transmitter to the receiver. Thus, it cannot be polarized. On the other hand, it has no problem propagating in a medium (air) that has no shear strength.

If a wave is polarized, then it must vibrate transversely from side-to-side. It can vibrate up-and-down (vertical polarization) or side-to-side (horizontal polarization). A violin string (Figure 2) yields a transverse standing wave. A wave must have a medium in which to vibrate (remember, this is the 19th century), and this as yet undetected medium was called the luminiferous æther. The æther must have zero shear strength. For example, starlight comes to us as vibrations on this

Figure 1. *James Clerk Maxwell founded the field of electromagnetics and is considered equal to Newton and Einstein.*

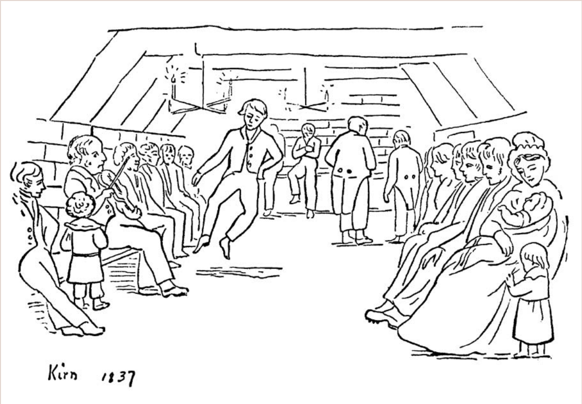

Figure 2. *Maxwell at about age six is interested in how the violin works, completely ignoring the dancer. The sketch is taken from a watercolor that was drawn from life.*

æther, and the earth simultaneously plows through the same space without resistance in the course of its orbit.

The problem is, how do you get a transversely undulating wave in a medium that has no shear strength? The wave theory of light isn't so perfect after all.

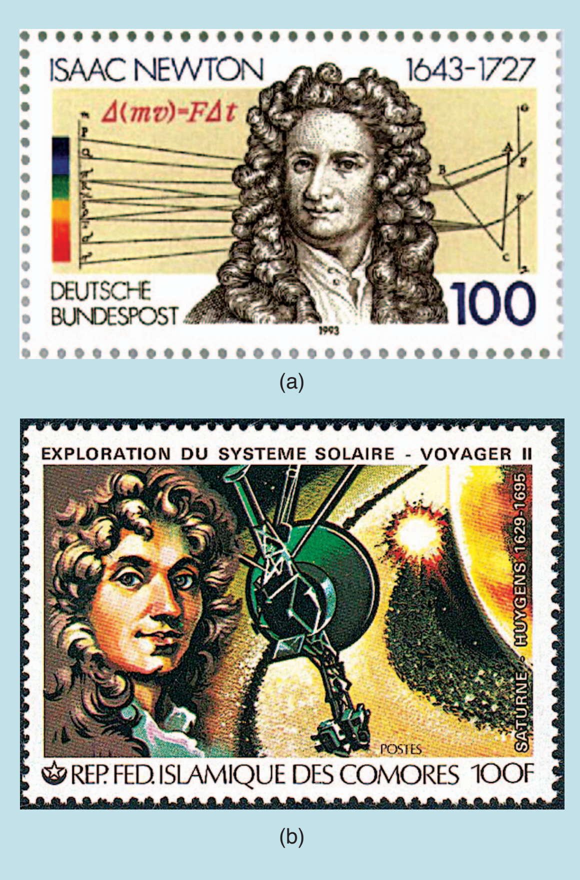

Figure 3. *(a) Newton and (b) Huygens proposed competing theories of light (particle versus wave) in the 18th century.*

Michael Faraday (Figure 5) then entered the picture. Just several months after Maxwell was born, Faraday was experimenting with magnetic induction. It seemed quite the reasonable thing to do. Given a coil of wire, electricity makes magnets. So, given that nature likes symmetry, shouldn't magnets make electricity? Such experiments had been attempted before and met with total failure. Magnets did not make electricity. Faraday met with the same failure.

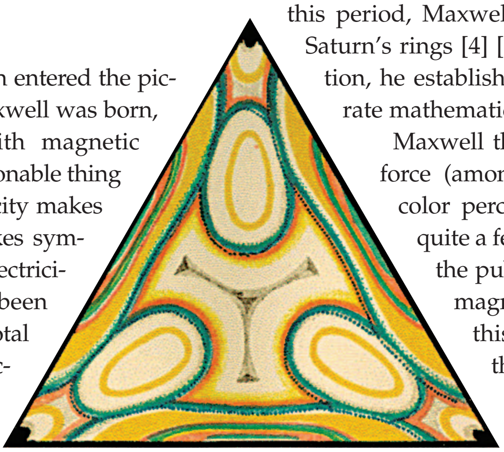

Figure 4. *As a teenager, Maxwell explored stress in unannealed glass using polarized light. Because light is polarized, it must be a transverse wave, a major difficulty for the wave theory of light.*

Wait a minute! What was that? Faraday noticed a slight flicker on the meter when he opened the switch on his experiment. Close the switch. Another flicker. He had it; turning the magnet on and off did the trick. Magnetic induction needs a *changing* current. Generators soon followed. Notice I have not mentioned the magnetic field; this concept did not yet exist.

Faraday, having come from a poor, humble family, did not have a university education and was unskilled in mathematics. He did, however, have superb intuition. He visualized induction as occurring due to the ebb and flow of the magnetic æther (different from the luminiferous æther) along magnetic "lines of force." He called these states of the magnetic æther the "electrotonic state." There were two problems: Faraday did not have the mathematical skills to follow up on his idea, and the concept of "action at a distance" was already very firmly accepted and validated.

First suggested by Priestly, then Cavendish, action at a distance was quantified by Coulomb using a torsion balance. According to this theory, the force between two charges varies with the inverse square of their distance. The same had been found to be true for magnets. True, there was some difficulty when things were moving or changing, but there were already some attempts to account for that, so why throw out all this work everyone had already done and use Faraday's complicated, imaginary lines of force?

Most importantly, Newton had already approved the action at a distance concept with his theory of gravity, even though he wasn't entirely happy with the lack of a mechanical attachment between gravitating bodies. Putting things in a Newtonian framework was important because, at this time, Newton was lord over all physics. If your work in physics did not somehow go back to $f = ma$ and Newton, you could forget about being treated seriously.

While Maxwell was in his mid-20s and a student at Cambridge, he began working with Faraday's lines of force. His work was interrupted when he left Cambridge for a professorship in Aberdeen. During this period, Maxwell worked on the composition of Saturn's rings [4] [see Figure 3(b)]. With its publication, he established his reputation as having first-rate mathematical ability.

Maxwell then worked on Faraday's lines of force (among many other things, including color perception and thermodynamics) for quite a few years, culminating in 1865 with the publication of his dynamical electromagnetic theory. The publication of this earth-shaking theory, perhaps the most significant event of the entire century, was greeted with . . . a great big yawn! But more on that later.

To see how Maxwell's theory developed, we explore three papers he published that described the state of his work. The first is "On Faraday's Lines of Force" [5], published in 1856 at age 24, just before Maxwell left Cambridge for Aberdeen:

> No electrical theory can now be put forth, unless it shews the connexion not only between electricity at rest and current electricity, but between the attractions and inductive effects of electricity in both states. Such a theory must accurately satisfy those laws, the mathematical form of which is known, and must afford the means of calculating the effects in the limiting cases where the known formulæ are inapplicable. In order therefore to appreciate the requirements of the science, the student must make himself familiar with a considerable body of most intricate mathematics, the mere retention of which in the memory materially interferes with further progress.

Here we see that the physics of the day recognized that electric current is simply static electricity in motion. They still did not know what form this electric fluid takes, or even if it is two kinds of fluid (positive and negative), one kind of fluid the absence of which appears to be the second kind, or some combination of the two.

We also see that Maxwell's goal was to link static-electric effects with current electric effects. He recognized that it would be difficult to reconcile the tremendous complexity of all existing partial results into one unifying theory.

He further stated "the results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis."

Maxwell here pointed out that the tradeoff of abstract mathematics is that you lose sight of reality. But with a physical model, you might achieve only a partial explanation. He stated that his investigation would be mathematical, but would remain closely tied to a physical model so as to enjoy the advantages of both approaches. Maxwell later completely dropped the physical hypothesis, but at this point the physical model was important. Newton and $f = ma$ had to be in there somewhere if Maxwell was to be taken seriously.

Maxwell, often citing the work of his close friend William Thomson (later Lord Kelvin), pointed out the frequent analogies found in nature:

> Yet we find that the mathematical laws of the uniform motion of heat in homogeneous media are identical in form with those of attractions varying inversely as the square of the distance. We have only to substitute *source of heat for centre of attraction, flow of heat for accelerating effect of attraction* at any point, and *temperature* for *potential* . . .

Maxwell used "analogical thinking" in this paper to explain electromagnetic forces in terms of fluid flow. Maxwell defined Faraday's lines of force as the path an inertialess object subject to the magnetic force would travel. Maxwell then visualized a tube along each of Faraday's lines of force. The tube was sized so that one unit of fluid always flows along one unit length of this tube in one unit of time. This means that at some distance from the magnet, the tubes become larger because the rate of flow (magnetic force) is smaller. Maxwell pointed out that the tubes have no empty space between them anywhere; the magnetic force is "flowing" like an incompressible fluid flow.

Historians acknowledge Maxwell to be the most important physicist of the 19th century, the equal of Einstein and Newton.

Maxwell stated that knowledge of this fluid flow over a closed surface then determines the (source-free) flow over the entire volume of the interior, according to the known laws of conduction. Magnetic force is determined by the differential pressure of this fluid flow, and magnetic permeability (modern term) is accounted for by the ease with which the fluid flows through the tubes. A similar model is used for electrostatic problems.

Maxwell hastens to caution his reader:

> By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena.

Maxwell emphasized that he in no way suggested that some kind of fluid flow actually causes the observed electromagnetic force. Rather, he tried out the analogy with fluid flow to see if it might be useful in guiding future investigations. The remainder of Maxwell's paper gradually developed a detailed mathematical theory of fluid flow and showed how it yields analogical results for electromagnetic forces.

Figure 5. *Michael Faraday discovered magnetic induction and proposed lines of force, inspiring Maxwell's electromagnetic theory.*

Incompressible fluid flow, imaginary or otherwise, cannot support a transverse wave such as light. In a second paper [6], "On Physical Lines of Force," published in 1862 at age 30, Maxwell retained the analog of a physical model:

In the century before Maxwell, the theories of Newton and Huygens competed for the mind of the natural philosopher, as physicists were then known.

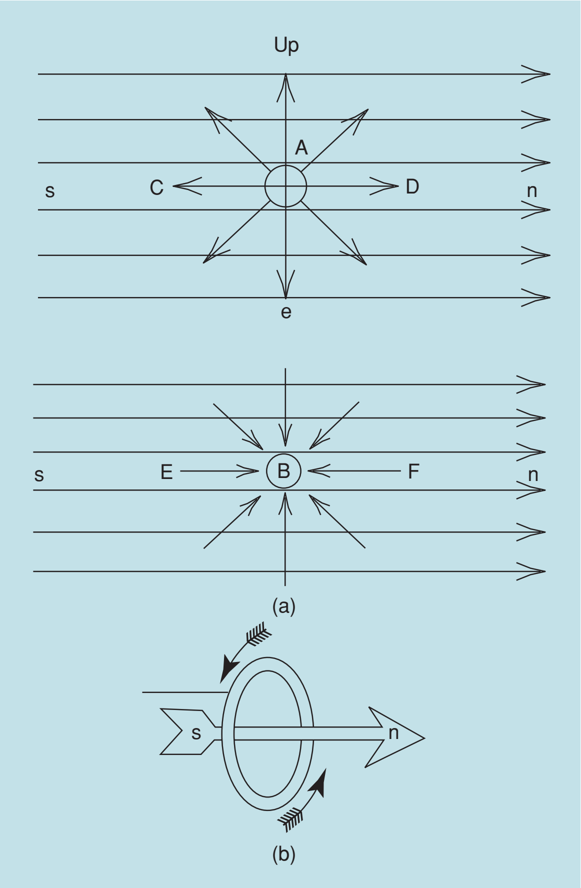

Figure 6. *(a) 1862: When lines of force are in the same direction, Maxwell shows that their force adds, pulling the north end of the compass needle to toward the north. (b) The right hand rule used by Maxwell.*

> I propose now to examine magnetic phenomena from a mechanical point of view, and to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed. If, by the same hypothesis we can connect the phenomena of magnetic attraction with electromagnetic phenomena and with those of induced currents, we shall have found a theory which, if not true, can only be proved to be erroneous by experiments which will greatly enlarge our knowledge of this part of physics.

The new mechanical model still made use of a fluid, but now the fluid was full of vortices. These vortices swirled around Faraday's lines of force. The lines of force are tension in the fluid, with the tension being created by the vortices. An example Maxwell provided (see Figure 6) shows the north end of a magnet [Figure 6(a)] embedded in a uniform "field of magnetic force" (the concept of a magnetic field still did not yet exist). The south pole of a magnet is embedded in the same field of force [Figure 6(b)]. When the lines of force (from the magnet and from the uniform field) are in the same direction, the vortices add (causing the tension, and thus the magnetic force) to increase. The opposite happens when the lines of force oppose. The net effect is that the magnet experiences a torque tending to make it point north like a compass.

Maxwell provided a detailed mathematical analysis of this mechanical model, showing that the velocity on the circumference of the vortices is proportional to the magnetic force and the density of the fluid is proportional to the "capacity of the medium for magnetic induction," i.e., permeability. Maxwell did not know what these vortices are, but he suggested they might be in some way formed by electric current.

Figure 6(b) is especially significant:

> We shall always mark by an arrow-head the direction in which we must look in order to see the vortices rotating in the direction of the hands of a watch. The arrow-head will then indicate the northward direction in the magnetic field . . .

This is the earliest published description and illustration I have seen of the right-hand rule, also used more recently in the IEEE's logo.

Maxwell pointed out that the vortex model does not explain electric current and that there is an obvious problem with the model:

> I have found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it.

To remedy this problem, Maxwell inserted "a layer of particles interposed as idle wheels," as shown in Figure 7. This layer of particles allows the vortices to all turn in the same direction. He also used these particles to model electric current as the net motion of the particles. Maxwell stated that these particles must be very small in size and mass with respect to the vortices. He also stated that they roll without slipping or touching within a molecule. But if they transfer from one molecule to another, they may experience resistance and generate heat. Viewed in today's world, this model is somewhat similar to the "sea of electrons" in a conductor, with the vortices being the atoms of the conductor and the idle wheels being electrons.

This mechanical system also models electrostatics, Maxwell's description of what we today call polarization charge:

> In a dielectric under [electrostatic] induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.

Maxwell then stated that this displacement of electricity is not a current because it remains bound to the vortices, but "it is the commencement of a current." The vortices were also given a degree of elasticity so that, when the displacement ends, the displaced electricity springs back into the molecule. This appears to be the beginning of Maxwell's displacement current, the critical term he added to what we today call Maxwell's equations.

Now that the vortices had elasticity, the mechanical model supported a transverse wave whose velocity, as calculated from the measured electrical and magnetic elasticity, agrees well with the measured velocity of light. Thus, Maxwell suggested that light and electromagnetic phenomena might be due to undulations of the same media. He did not yet quite suggest that light itself might actually be electromagnetic.

Maxwell's key paper [7], titled "A Dynamical Theory of the Electromagnetic Field," was read (presented at a meeting) at the end of 1864, when Maxwell was 33. Here he presented a complete electromagnetic theory. However, it was not the familiar four equations. First, Maxwell did not have the use of modern vector algebra, which was developed by Gibbs (famous for Gibbs phenomenon) after Maxwell's death (see Figure 8). In this paper, Maxwell wrote out the vector differential equations as triplets of scalar equations in Cartesian coordinates. Later, in the third edition of his treatise [8], he used "quaternions," making matters even worse [9]. At one point in this paper, Maxwell did use ∇^2, but only as a notational convenience.

Second, Maxwell took magnetic vector potential (this is Faraday's "electrotonic state") as primary. Magnetic and electric fields were secondary. Heaviside commented [9]: "I never made any progress till I threw all the potentials overboard, and made E and H the objects of attention . . . ."

Heaviside also noted that the modern duplex form (which shows the symmetry between E and H, developed by Heaviside) frankly:

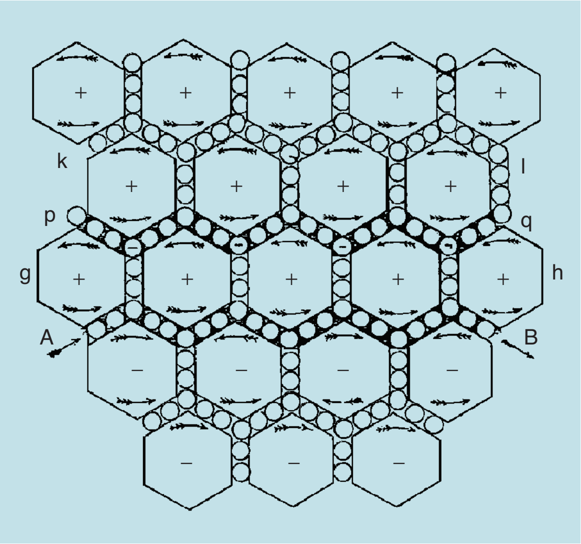

Figure 7. *Maxwell introduces a layer of small, frictionless particles acting as idle wheels to keep the vortices of his electromagnetic medium from directly touching each other. These particles can flow, for example, going in at A and out at B, creating electrical current.*

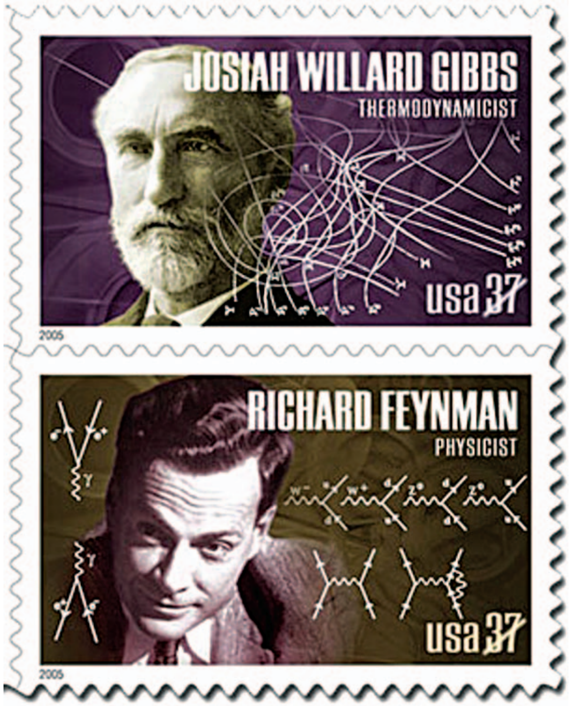

Figure 8. *It is not well known that Gibbs developed our modern vector algebra, critical in EM theory. He is also the father of modern thermodynamics and had numerous communications with Maxwell. Richard Feynman helped develop Quantum Electrodynamics, the successor to Maxwell's EM theory. These stamps are currently available in US post offices.*

> . . . brings to light many useful relations which were formerly hidden from view by the intervention of the vector-potential and its parasites.

Maxwell presented 20 equations in 20 variables in this paper. Heaviside put them into their modern form by mathematical manipulation. Hertz (see Figure 9) independently derived the same modern form by applying an infinite series of local corrections to the action at a distance model. Both discarded potentials as primary. Ironically, physicists are now returning to treating potentials as primary [10].

While Maxwell was in his mid-20s and a student at Cambridge, he began working with Faraday's lines of force.

Figure 9. *Heaviside (not illustrated) and Hertz put Maxwell's equations into their modern form. Maxwell never saw what we call Maxwell's equations.*

In this paper, Maxwell stated:

> The theory I propose may therefore be called a theory of the *Electromagnetic Field*, because it has to do with the space in the neighbourhood of the electric or magnetic bodies . . .

He also noted that:

> The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions.

This suggests, to me, that Maxwell used the term "field" as a region of interest, as in a "battlefield." I emphasize that this is a personal observation; professional historians are likely to have differing viewpoints. I do not know when the concept of a vector field was formally introduced. However, we do know that the formal framework for vector algebra was not introduced until after Maxwell's death, as mentioned above.

My personal impression is that Maxwell fully realized the importance of his introduction of displacement current because he presented it as the first triplet of equations in his theory. However, with characteristic Maxwell modesty, he made no mention of its importance. In modern terms, this first set of three scalar equations in Cartesian coordinates states that total current is the conduction current plus the displacement current, shown here exactly as they appeared in his paper:

$$p' = p + \frac{df}{dt}$$
$$q' = q + \frac{dg}{dt}$$
$$r' = r + \frac{dh}{dt}.$$

Historians emphatically state that Maxwell did not add the displacement current to achieve symmetry in his equations. In the way Maxwell presented his 20 equations, symmetry is anything but clear. In fact, Maxwell did not include the symmetrical magnetic charge or current; that addition was effected by Heaviside. Historians also state that it is unknown exactly how he was inspired to insert the displacement current, but it seems to me the bound charge analogy, as described above, had something to do with it. Maxwell did mention the problem of propagation in "so called vaccua," and pointed out that the EM forces are not transmitted by matter, but must be transmitted by some sort of æther that exists even in a vacuum.

It is in this paper that Maxwell finally, based on the near equality of the measured and calculated velocity of light, concluded:

> This velocity is so nearly that of light, that it seems we have strong reason to conclude that light itself (including radiant heat, and other radiations if any) is an electromagnetic disturbance in the form of waves propagated through the electromagnetic field according to electromagnetic laws.

Maxwell did show playfulness here by further stating, "the only use made of light in the experiment was to see the instruments," when measuring the permeability and permittivity.

The most striking thing about this paper is that Maxwell had completely abandoned the mechanical model. While there are still mechanical analogies and his theory yields EM forces as a result, lack of a mechanical model means that Newton, the lord of all physics, had nothing at all to do with the inner workings of this theory. Politically, this was very bad news; but Maxwell never was political.

To make matters worse, Maxwell was extremely modest. For example, in 1870, he was president of

Section A of the British Association for the Advancement of Science. His inaugural address was published in the second volume of the new journal *Nature*. Maxwell used most of the lecture to promote a somewhat ridiculous (by today's standards) atomic theory proposed by his close friend, Lord Kelvin, rather than to promote his own EM theory. At the very end, he did mention, "Another theory of electricity which I prefer . . .," failing to even take personal credit for his own theory.

Freeman Dyson points this out in an essay [11] in which he also describes the case of physicist Michael Pupin. Pupin traveled from America to Cambridge in 1883 to learn Maxwell's theory from Maxwell himself, only to find that Maxwell had passed away four years earlier. Then he found that there was no one in all of Cambridge who could teach Maxwell's theory. He finally traveled to Germany and learned Maxwell's EM theory from Helmholtz. Pupin returned to America, where he taught generations of students EM theory at Columbia.

Today we often consider things like electric and magnetic fields to be real. This is not the case. The fields are purely abstract mathematical constructs that allow us to predict things we can actually perceive.

As Dyson points out, we easily perceive and measure things like energy and distance. We can deal with electrical energy density, proportional to E^2, in Joules per cubic meter. Measure the Joules with a calorimeter. Measure a cubic meter with a stick. But how do we measure electric field directly? You first need a square root of a calorimeter. Then, what kind of stick do you use to measure the square root of a cubic meter? We can only infer Maxwell's abstract electric field from what we can directly measure, as in Joules, Newtons, and meters.

If we cannot directly measure or sense electric or magnetic fields, what good are they? As Dyson points out, once we have solved for these fields, we can form quantities that we actually can sense and measure, like E^2, H^2, or $E \times H$. We have exactly the same situation in quantum electrodynamics (QED, of which Maxwell's theory is a special case), as most capably described by Richard Feynman [12] (see Figure 8).

It took 20 years for Maxwell's theory to be recognized for what it is. This is, at least in part, due to its complexity (why should we bother with all this complicated "field" stuff when action at a distance works just fine?). This is also due to its lack of an underlying mechanical model (Lord Kelvin went to his grave convinced that Maxwell's theory was not correct for this reason). Lastly, it is due to Maxwell's own modesty.

In the same essay, Dyson suggests that physics was set back 20 years because of Maxwell's modesty. It was not the failure to recognize the importance of Maxwell's equations, per se, that set back physics; rather, it was the failure to recognize the new worlds opened by Maxwell's invocation of abstract mathematics with no link to Newton and no pretense to actually understand the underlying reality. According to Dyson:

> The ultimate importance of the Maxwell theory is far greater than its immediate achievement in explaining and unifying the phenomena of electricity and magnetism. Its ultimate importance is to be the prototype for all the great triumphs of twentieth-century physics. It is the prototype for Einstein's theories of relativity, for quantum mechanics, . . . and for the unified theory of fields and particles that is known as the Standard Model of particle physics. All these theories are based on the concept of dynamical fields, introduced by Maxwell in 1865.

Maxwell then states that this displacement of electricity is not a current because it remains bound to the vortices, but "it is the commencement of a current."

So this is Maxwell's legacy, his concept of the mathematically abstract field that freed physics from the constraining womb of Newtonian mechanics and set the stage for all the great advances in 20th century physics.

References

[1] New DMLs for 2005–2007 named [Online]. Available: http://www.mtt.org

[2] L. Campbell and W. Garnett, *The Life of James Clerk Maxwell*, 2nd ed. London: Macmillan, 1884.

[3] J.C. Maxwell Biography [Online]. Available: http://www.sonnetsoftware.com/bio/maxwell.asp

[4] S.G. Brush, C.W.F. Everitt, and E. Garber, *Maxwell on Saturn's Rings*. Cambridge, MA: MIT Press, 1983.

[5] J.C. Maxwell, "On Faraday's lines of force," *Trans. Cambridge Philosoph. Soc.*, vol. X, Part I. 1855. Repr. *The Scientific Papers of James Maxwell*, 1890. New York: Dover, pp. 155–229.

[6] J.C. Maxwell, "On physical lines of force," *Philosoph. Mag.*, vol. XXI, Jan–Feb. 1862. Repr. in *The Scientific Papers of James Maxwell*, 1890. New York: Dover, pp. 451–513.

[7] J.C. Maxwell, "A dynamical theory of the electromagnetic field," in *Royal Soc. Trans.*, vol. CLV, Dec. 8, 1864. Repr. in *The Scientific Papers of James Maxwell*, 1890. New York: Dover, pp. 451–513.

[8] J.C. Maxwell, *A Treatise on Electricity and Magnetism*, 3rd ed. New York: Dover, 1954.

[9] P.J. Nahin, *Oliver Heaviside: Sage in Solitude*. New York: IEEE Press, 1987. Repr. with a new introduction. Baltimore, MD: Johns Hopkins Univ. Press, 2002.

[10] F. Gronwald and J. Nitsch, "The structure of the electromagnetic field as derived from first principles," *IEEE Antennas Propagat. Mag.*, vol. 43, pp. 64–79, Aug. 2001.

[11] F. Dyson, "Why is Maxwell's theory so hard to understand?" in *James Clerk Maxwell Commemorative Booklet*, *Fourth Int. Congress Industrial and Applied Mathematics*, Edinburgh, Scotland, July 1999.

[12] R.P. Feynman, QED: *The Strange Theory of Light and Matter.* Princeton, NJ: Princeton Univ. Press, 1985.