Application of the Wheeler Incremental Inductance Rule for Robust Design and Modeling of MMIC Spiral Inductors

Grant A. Ellis

Department of Electrical and Electronic Engineering
Universiti Teknologi Petronas, 31750 Tronoh, Perak Darul Ridzuan, MALAYSIA
grant_ellis@petronas.com.my

Abstract: In electromagnetic modeling of MMIC inductors, a fine grid and several sheets are used to accurately model the current distribution and determine the resistance. Sonnet™ is used to accurately model the 3D characteristics of thick conductors such as loss and electromagnetic effects of physically thick metal. An accurate procedure based on the Richardson extrapolation method is used to extract the resistance values without long computation time. In this paper, it is shown that the series resistance of an MMIC inductor can be used as a figure of merit for the robustness of the inductor against etching variations in line width during fabrication. The results also show that circular inductors have less inductance variation than rectangular inductors.

Keywords: Inductors, MMICs, Microwave circuits, Modeling.

1. Introduction

During fabrication of MMIC inductors, variations in line width or line thickness may occur. This can be due to variations in the etch rates during wafer processing. The effect of these processing variations can result in detuning of MMICs and lowering circuit yield. As development cycles for wireless products accelerate, design techniques for planar spiral inductors for robust performance in the presence of these process variations in the fabrication of MMICs become necessary.

The Wheeler incremental inductance rule shows that the total change in inductance due to the variation of all conductor surfaces is proportional to the resistance [1-4]. In our earlier paper [5], the inductance change due to variation in the width and thickness of the inductor line was approximated for circular inductors. Here, circular inductors are assumed to have a nearly continuous current distribution while rectangular inductors due to current crowding in the corners have a mostly discontinuous distribution. The simulation results show that circular inductors have lower resistance than the corresponding rectangular inductors with nearly the same dimensions and inductance values. This is not surprising considering that the rectangular inductor experiences current crowding in the corners resulting in higher resistance. The circular inductors however, also show less change in inductance due to variation in line width than the corresponding rectangular inductors. This result is implied by the incremental inductance rule.

This paper begins with a simple example showing how the Wheeler Incremental Inductance rule is utilized to calculate the resistance in parallel conductors. Sonnet em [6] is used to calculate the scattering parameters for several inductors. The resulting equivalent circuit model parameters are extracted and the series resistance values are used to estimate the change in inductance due to variation in line width. Using this method, accurate calculation of the resistance becomes necessary. Multiple sheets and a fine discretization grid must be used to get high accuracy. A convergence test is used to verify the extracted model parameter values.

2. Theoretical Development

Consider two parallel round wires having currents flowing in opposite directions. Each wire has radius *a,* and the center-to-center spacing is *2b* as shown in Fig. 1 below:

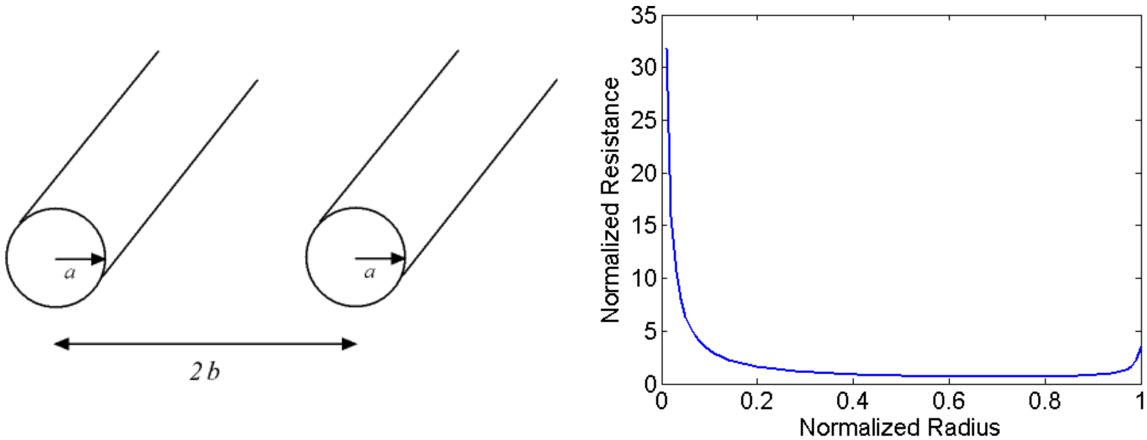


Fig. 1 Resistance for two parallel circular conductors.

The external inductance for the two-wire pair is: $L = \frac{\mu_0}{\pi} cosh^{-1}\left(\frac{b}{a}\right)$ (1)

Using the incremental inductance rule, the resistance for the two wires becomes [7]:

$$R = \frac{R_s}{\mu_0}\frac{\partial L}{\partial a} = \frac{R_s}{\pi a}\frac{1}{\sqrt{1 - a^2/b^2}} \quad (2)$$

For the parallel wires, the resistance is minimum when $a = \sqrt{2}\, b$. For $a << b$, the resistance is equal to twice the resistance of each wire. For $a \sim b$, the resistance quickly increases due to the proximity effect (Fig. 1). According to the incremental inductance rule, circular inductors which have no corners and thus a lower resistance than rectangular inductors should also have less variation in inductance due to changes in line width or thickness [5].

3. Numerical Results

Sonnet is a full wave electromagnetic simulator for planar microwave structures and is a tool of choice due to high accuracy requirements. Microstrip inductors with line width, w = 5 and 10 μm and spacing, s = 5 μm on Gallium Arsenide (GaAs, ε_r = 12.9) with a substrate thickness of h = 100 μm and metal thickness of t = 2 μm are simulated. The microstrip conductor is Gold and for simplicity the loss tangent for the GaAs substrate is assumed to be zero. The thick metal approximation in Sonnet is used to accurately model the 3D characteristics of thick conductors such as loss and electromagnetic effects of physically thick metal such as coupling between closely spaced conductors. For circular inductors, a conformal type mesh is used. For rectangular inductors, the staircase type fill is used.

For accurate EM analysis, a procedure is first carried out to determine the number of sheets and grid spacing necessary. The microstrip circular inductors with $N = 3$ and 4 turns are analyzed for increasing number of sheets and finer grid spacing. The number of sheets is increased until the extracted resistance and inductance values of the inductor converge [8]. The Pointer Robust Optimization in Microwave Office is used to extract the series inductance, resistance, parallel capacitance, and shunt capacitances to ground [5]. This optimization uses multiple search methods to fit the simulated data from Sonnet to the inductor equivalent circuit model [9]. At 16 GHz, the $t = 2$ μm thick Gold inductor is about 3 skin depths thick.

Tables 1 and 2 show the extracted resistance, inductance, and parallel capacitance values of two circular inductors having $w = 10$ μm and $N = 3$ and 4 turns. The extrapolated values using the 3-point Richardson's extrapolation technique are included. Using the earlier work by Richardson [10], a more accurate solution is achieved without a very fine discretization. The convergence ratio gives a measure of the goodness of the extrapolation [11 - 13]. For true monotonic convergence, the convergence ratio is unity. As shown in these Tables, the inductance and parallel capacitance values converge very quickly. The resistance values, however, converge more slowly. The extracted resistance and inductance values converge to within 1 % of their final value for 15 sheets using a 0.5 μm grid. To conserve simulation time, a 0.5-μm minimum layout grid size is used for 10 μm wide lines. Also, 15 sheets or about five sheets per skin depth at 16 GHz are used to model the thick conductor. EM simulations were carried out with Sonnet, Release 12 [6] on a workstation using a dual Intel Xeon 2.66 GHz CPU (8 cores) and 16 GByte of RAM.

Table 1a: Convergence for Circular Inductor Resistance.
(w = 10 μm, N = 4 turns)

Grid Size / Number of Sheets	1 μm (10)	0.714 μm (14)	0.5 μm (20)	Extrapolated Value	Convergence Ratio
10	5.022 Ω	5.37 Ω	5.643 Ω	**5.9629 Ω**	0.98
15	5.033 Ω	5.451 Ω	5.753 Ω	**6.0954 Ω**	0.99
20	5.032 Ω	5.496 Ω	5.78 Ω	**6.08 Ω**	0.99

Table 1b: Extracted Inductance and Parallel Capacitance values for Circular Inductor.
(w = 10 μm, N = 4 turns)

Grid Size / Number of Sheets	1 μm (10)	0.714 μm (14)	0.5 μm (20)
10	1.512 nH 0.014 pF	1.511 nH 0.0141 pF	1.515 nH 0.0141 pF
15	1.512 nH 0.0141 pF	1.511 nH 0.0141 pF	1.515 nH 0.0142 pF
20	1.512 nH 0.0141 pF	1.513 nH 0.0141 pF	1.516 nH 0.0142 pF

Using the procedure described above, the simulated data for four circular and rectangular inductors from Sonnet are used to extract the equivalent inductance and resistance values versus variation in line width, Δw at f = 16 GHz. The inductors have N = 3 and 4 turns, with w = 5 and 10 μm and s = 5 μm. The rectangular and circular inductors also have nearly the same nominal inductance value, e.g. 0.81 nH for N = 3 turns, w = 5 μm and 1.45 nH for N = 4 turns, w = 5 μm. Fig. 2a and 2b show the extracted resistance and inductance values vs. over- and under-etch of the line width, Δw for N = 3 turns, w = 5 and 10 μm. Fig. 2c and 2d show the extracted resistance and inductance values vs. amount of over- and under-etch of the line width, Δw for N = 4 turns, w = 5 and 10 μm. In each case, the center line to line spacing, $w + s$, is held constant and the extracted inductance values vary linearly with Δw. The nominal resistance vs. the total change in inductance for the circular and rectangular inductors is shown in Table 3. For N = 3 turns and w = 10 μm, the variation in extracted inductance due to variation in line width is +/- 10.7 % for the rectangular inductor and +/-5.5 % for the circular inductor. For N = 4 turns and w = 10 μm, the variation in extracted inductance due to variation in line width is +/-9.5 % for the rectangular inductor and only +/-5 % for the circular inductor.

Table 2a: Convergence for Circular Inductor Resistance.
(w = 10 μm, N = 3 turns)

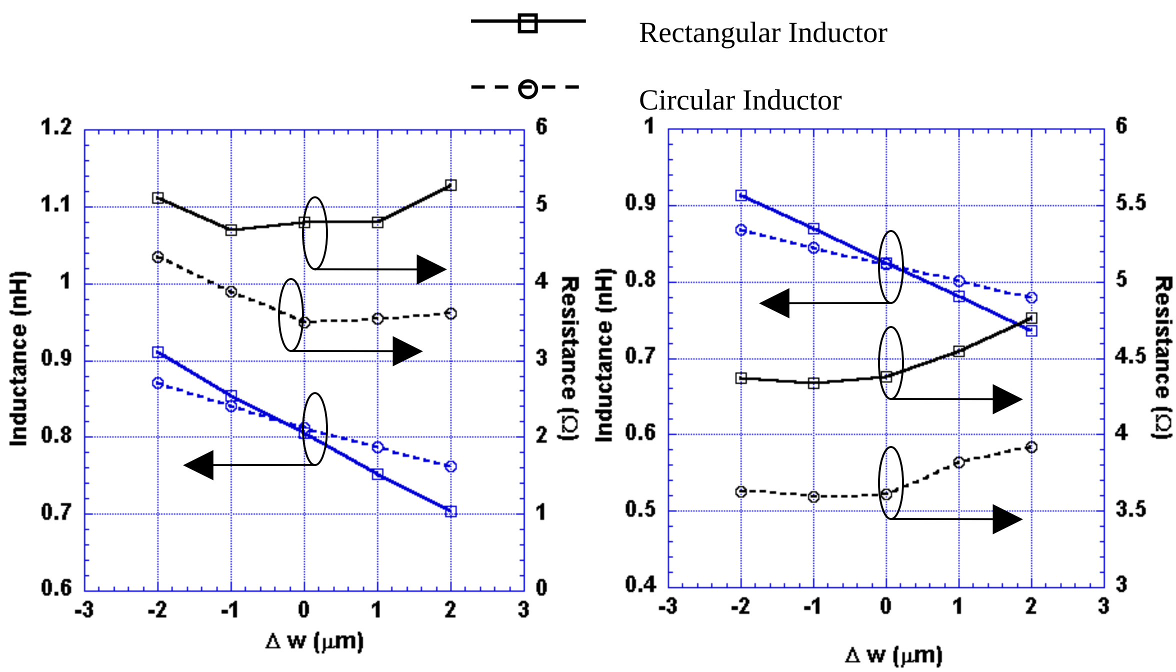
Grid Size / Number of Sheets	1 μm (10)	0.714 μm (14)	0.5 μm (20)	Extrapolated Value	Convergence Ratio
10	2.958 Ω	3.169 Ω	3.328 Ω	**3.511 Ω**	0.99
15	2.992 Ω	3.232 Ω	3.407 Ω	**3.606 Ω**	0.99
20	3.03 Ω	3.242 Ω	3.426 Ω	**3.649 Ω**	0.98

Table 2b: Extracted inductance and parallel capacitance values for Circular Inductor.
(w = 10 μm, N = 3 turns)

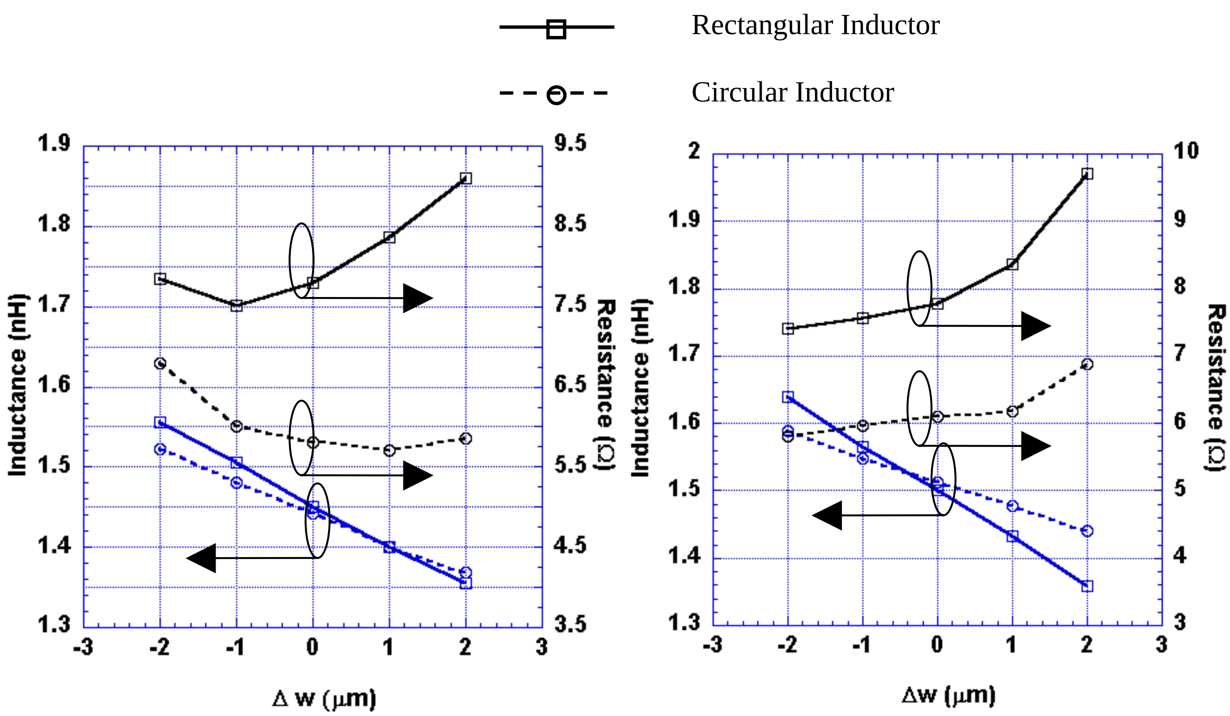
Grid Size / Number of Sheets	1 μm (10)	0.714 μm (14)	0.5 μm (20)
10	0.8237 nH 0.0113 pF	0.8223 nH 0.0113 pF	0.8249 nH 0.0113 pF
15	0.8244 nH 0.0116 pF	0.8227 nH 0.0113 pF	0.8246 nH 0.0114 pF
20	0.8246 nH 0.0114 pF	0.8233 nH 0.0113 pF	0.8253 nH 0.0113 pF

Table 3: Inductor Resistance vs. change in Inductance.

	Resistance Circular	Resistance Rectangular	Δ L Circular	Δ L Rectangular
N = 3, w = 5 μm	3.5 Ω	4.8 Ω	+/- 6.4 %	+/- 13 %
N = 4, w = 5 μm	5.8 Ω	7.8 Ω	+/- 5.3 %	+/- 7 %
N = 3, w = 10 μm	3.6 Ω	4.4 Ω	+/- 5.5 %	+/- 10.7 %
N = 4, w = 10 μm	6.1 Ω	7.8 Ω	+/- 5 %	+/- 9.5 %



(a) N = 3 turns, w = 5 μm (b) N = 3 turns, w = 10 μm



(c) N = 4 turns, w = 5 μm (d) N = 4 turns, w = 10 μm

Fig. 2 Variation in Inductance and Resistance vs. over- and under-etch.

4. Conclusion

The extracted inductance and parallel capacitance values for planar microstrip inductors using electromagnetic analysis converge quickly requiring only a 1 um grid and a few sheets. The extracted resistance values, however, are shown to converge more slowly using a convergence analysis technique and require at least a 0.5 um grid and up to five sheets per skin depth to accurately capture the cross sectional current distribution of the conductor. An extrapolation procedure is used to accurately extract the resistance values without lengthy computation time. The Wheeler incremental inductance rule provides a quantitative relationship between the resistance and inductance in electrical circuits. Results show that circular inductors are more tolerant than rectangular inductors to variations in line width as anticipated by the incremental inductance rule. Circular inductors show about one half of the variation in the inductance value due to over- or under-etching than corresponding rectangular inductors having about the same inductance value. Finally, the results show that an optimum line width, w, can be found for minimum resistance in planar spiral inductors.

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