Application Notes

An Investigation of Microstrip Conductor Loss

■ James C. Rautio

When asked how microstrip conductor loss varies with frequency, most high frequency designers immediately answer, "With the square root of frequency." In other words, if frequency is quadrupled, then loss is doubled. We have a nice simple answer for a nice simple question; however, this simple answer does not withstand scrutiny. Take any microstrip transmission line and an ohm meter. While the measured resistance is small, it is not zero. However, the square-root-of-frequency rule predicts zero resistance at zero frequency. I call this the "ohm-meter paradox."

Most experienced high frequency designers are aware of the ohm-meter paradox, but they often do not know why or at what frequency the square-root rule fails. This question arose on an Internet forum (http://rf.rfglobalnet.com/forums/General/Forum.asp, thread 895) and inspired the research presented in this article. All numerical electromagnetic (EM) analyses described in this article can be duplicated with the Sonnet Lite planar EM solver, available for free download at http://www.sonnetusa.com.

High Frequency Conduction

The result of the Internet discussion and subsequent research showed that there are three, not just two, distinct frequency regions of interest. The high frequency region is most familiar to the high frequency designer. Loss increases with the square root of frequency due to the "skin effect." Skin effect occurs at high frequency for good conductors and is due to electric current being restricted to the surface of the conductor. As the frequency increases, this skin-effect layer of current becomes thinner. As the current becomes more and more confined to the surface, resistance increases.

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Skin depth is given by

$$\delta = \frac{1}{\sqrt{\pi \mu \sigma f}} \qquad (1)$$

where

δ = skin depth (m)

μ = conductor magnetic permeability ($\mu_0 = 4\pi \times 10^{-7}$) (H/m)

σ = bulk conductivity (S/m)

f = frequency (Hz).

We can see from the above that if frequency quadruples, then skin depth is cut in half. Since the current can now flow in only half of the original cross-section, resistance doubles.

Notice μ in the equation. Designers sometimes forget that magnetic permeability has the same importance as frequency. If a magnetic metal (like nickel) is used, a large increase in skin-effect resistance should be expected.

Also well known theoretically is something called the "edge singularity." Any current flowing on a good conductor tends to flow close to and parallel to any sharp edge. In fact, if the edge is infinitely sharp, infinite current density flows parallel to and exactly on the edge. This does not happen in practice, because the infinite electric fields would kick the atoms off the conductor edge and the edge would no longer be infinitely sharp. This is called electromigration.

A good intuitive understanding of the edge singularity is provided by the following thought experiment. Imagine placing electrostatic charge on a microstrip conductor. The electrons in the electrostatic charge repel and push each other to opposite edges of the line. This represents the minimum potential energy configuration. High frequency is similar; just imagine the electrons now moving back and forth. (This last step is called a "quasi-static approximation." Experts will recognize I have simplified matters.)


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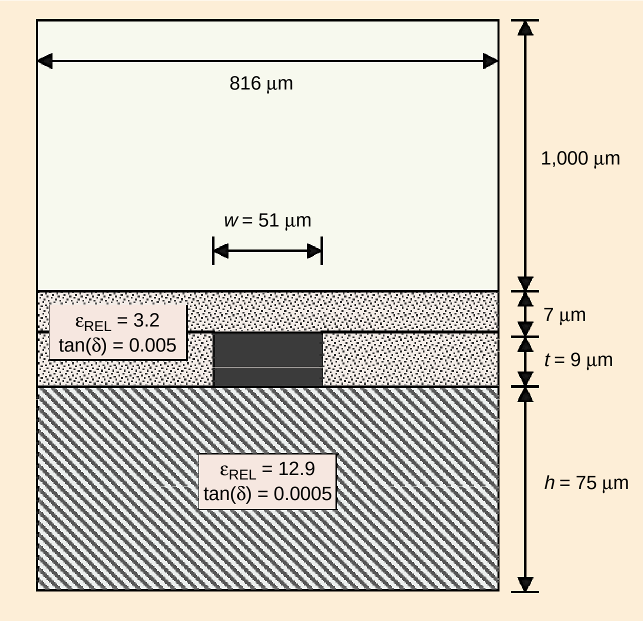

Figure 1. *Transmission-line geometry used for analysis. The substrate is GaAs with polyimide passivation. In the actual line, the polyimide is 7-µm thick everywhere. Loss tangent of the polyimide is 0.005, and the GaAs is 0.0005. Drawing not to scale.*

In summary, skin effect concentrates current near the conductor surface, increasing resistance. In addition, the edge effect further restricts most current to the edges of the conductor, increasing resistance even more. This is how high frequency loss becomes large.

High Frequency Transition

Let's make a first attempt to explain the ohm-meter paradox. How can we have loss at zero frequency, when the square-root-of-frequency rule says we should have no loss? Skin depth is the obvious answer. At zero frequency, skin depth is much larger than conductor thickness, and current is uniform through the entire thickness of the line. If we lower the frequency, the cross-section in which the current flows can no longer increase. Thus, at low frequency, loss should be constant with frequency.

The frequency at which a conductor transitions from electrically thick (high frequency) to electrically thin (medium frequency) is

$$f_{c2} = \frac{4}{\pi \mu \sigma t^2} \tag{2}$$

where f_{c2} is the critical frequency (Hz) at which the conductor thickness equals twice the skin depth. We use twice the skin depth because there is skin effect current on both sides of the conductor.

Verification by Numerical Experiment

Let's check to see what happens in a quick numerical simulation. We simulated a section of a line made of gold for which we have measurements. Details of the analysis geometry are in Figure 1. The actual transmission line has 7 µm of polyimide everywhere.

For high accuracy,For this analysis, the microstrip line is modeled as two infinitely thin sheets of conductor. One conductor sheet is placed directly on the surface of the substrate to represent the bottom side of the actual metal. The other conductor sheet is placed at the top of the actual metal. For high frequency, when the conductor is thick with respect to skin depth, the two infinitely thin sheets of conductor represent the two skin depth generated sheets of current in the actual conductor. At low frequency, the current simply splits equally between the two infinitely thin sheets of conductor. Details of how loss is modeled in Sonnet are provided in a sidebar in this article. this line is subsectioned 64-cells long and 16-cells wide. Analysis time, including deembedding, is 7 seconds per frequency on a 400 MHz Pentium. A special option (Output Files->.lct file name) is used to automatically synthesize the lumped transmission-line parameters, in addition to S-parameters. A logarithmic frequency sweep is specified (Complex Sweep->Edit->Add) from 1 MHz to 20 GHz.

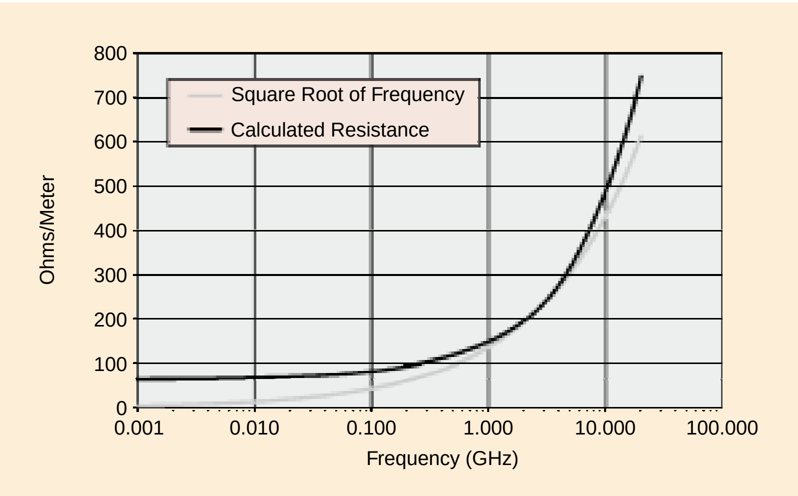

Figure 2. *Calculated resistance per unit length shows significant variance from the standard square-root-of-frequency model.*

Figure 2 shows the calculated resistance per meter for the microstrip line. It behaves very much as expected. Loss increases at high frequency and is constant at low frequency. A curve for a square-root-of-frequency model is also included. The unexpected behavior (relative to the square-root-of-frequency model) above 5 GHz is discussed later.

For low frequency behavior, everything seems fine. The critical frequency, f_{c2}, is 363 MHz. At this frequency, loss is definitely becoming more constant with frequency. It is now very tempting to conclude that we understand microstrip loss and end our investigation.

However, instead of looking at Figure 2 with the attitude that we are correct and understand what is going on, let's look at Figure 2 critically. The critical observer can see that loss does not become completely independent of frequency until below 10 MHz, over one order of magnitude lower in frequency. Perhaps we should consider the possibility that we do not yet have a complete understanding.

where

f_{c1} = first critical frequency (Hz)

R = resistance per unit length (= $1/(\sigma wt)$, w = width(m), t = thickness(m)) (Ω/m)

Visual Verification

When something unexpected happens in high frequency design, diagnosis is often best performed by viewing the current distribution. Figure 3 shows the current distribution on the microstrip line of Figure 1 at 400 MHz, 40 MHz, and 4 MHz. At 400 MHz, the microstrip line still shows a strong edge singularity even though it should be starting to disappear. The current in one cell at the edge is 2.9 times that at the center (cell size = 0.4 µm). Even at 40 MHz, the edge singularity is still there (edge to center ratio 1.13). Finally, at 4 MHz, the edge singularity is gone (edge to center ratio 1.003). A different scale is used in Figure 3 for the 400 MHz distribution because of the large dynamic range.

We can now see that two transitions must happen in order to resolve the ohm-meter paradox. First, the frequency must become low enough that the microstrip line is thin compared to skin depth. This occurs at f_{c2} = 363 MHz. Second, the edge singularity must disappear, leaving a uniform current distribution across the width of the line. Now, the line acts like a simple resistor with current flowing uniformly through the entire cross-section of the conductor.

At what frequency does the edge singularity disappear? My first guess, and that of others I asked, is that when the conductor width (not thickness) is small compared to skin depth, then the edge singularity should disappear.

This is a nice sounding hypothesis, but does it withstand critical quantitative scrutiny? For this microstrip line, the conductor width equals twice the skin depth at 11 MHz. This could be consistent with both Figure 2 and Figure 3, and, once more, we might be tempted to end this line of investigation.

However, the skin-depth-line-width hypothesis fails for other transmission line geometries. For example, a 2-µm thick, 0.635-mm wide line on 0.635-mm thick Alumina also demonstrates nearly constant resistance below 10 MHz. However, the skin-depth-line-width hypothesis predicts the transition should occur around 70 kHz! This hypothesis must be discarded.

The second hypothesis we considered is more robust. If we assume the transition frequency is that frequency for which the resistance per unit length equals the inductive reactance per unit length, we have

$$f_{c1} = \frac{R}{2\pi L} \tag{3}$$

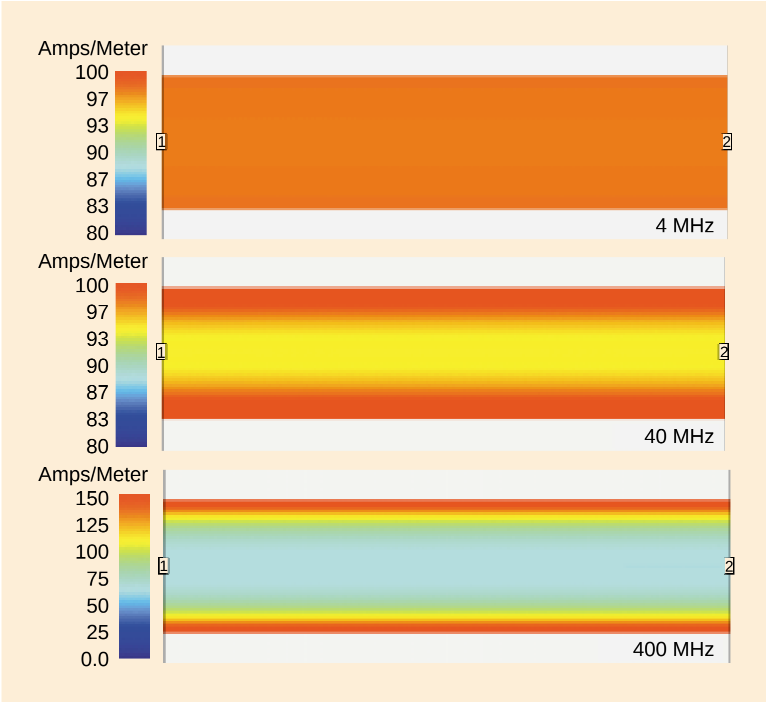

Figure 3. *Edge singularity is still clearly present at 400 MHz (bottom), and even at 40 MHz. However, it is gone at 4 MHz (top). This suggests that at around 40 MHz, the current distribution encounters a second transition. Port 1 is excited by a 1 V source in series with 50 Ω. Port 2 is terminated in 50 Ω. Note different scales.*

```
!<FTYP NET

DIM        ! Define units for circuit parameters.
  FREQ     GHZ

CKT        !Analyze 215.25 um long line, cascade 32 times.
           GEO 1 2 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 2 3 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 3 4 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 4 5 6over32mm.geo OPT=vd CTL=ctl.an
           :
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           GEO 29 30 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 30 31 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 31 32 6over32mm.geo OPT=vd CTL=ctl.an
           GEO 32 33 6over32mm.geo OPT=vd CTL=ctl.an
           DEF2P 1 33 line

FILEOUT !Output the 2 port parameters for 'line'.
           line TOUCH 6mm.s2p S MA R 50

FREQ
           FREQ SWEEP 0.05 20.05 .1
```

Figure 4. *The Sonnet netlist analysis capability is used to cascade the 215.25-µm-long line 32 times to yield analysis results for a 6,888-µm-long line. Each GEO line launches an electromagnetic analysis. However, Sonnet recognizes that each one is identical and only one EM analysis is performed with the result used 32 times.*

The Sonnet Model of Conductor Loss

All computer analysis requires an abstraction of a model from physical reality. As such, computer analysis is always an approximation to the reality being modeled. Details of the two-sheet Sonnet loss model are provided here. This allows the reader to judge the appropriateness of the model and allows the reader to duplicate the model if desired.

At high frequency, the physical conductor being modeled is thick with respect to skin depth. As such, there are two thin sheets of current flowing on the conductor. One sheet of current flows on the bottom side of the conductor. The other sheet of current flows on the top side of the conductor.

Planar analyses, including Sonnet, model infinitely thin sheets of current. For high frequency, abstraction of a model appropriate for Sonnet analysis is simple. We model the conductor as two infinitely thin sheets of current. One sheet is placed at the bottom of the thick conductor. The other sheet is placed at the top.

At low frequency, current flows uniformly through the entire cross-section of the line. Current in the abstracted model is split evenly between the top and bottom.

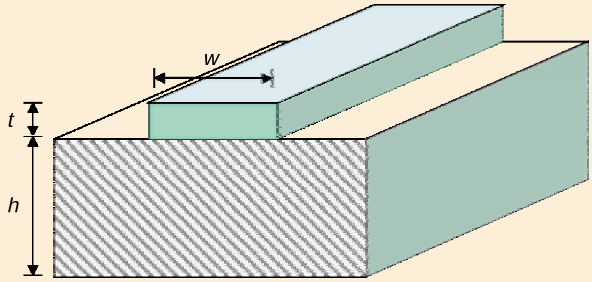
In order to determine the error involved in using this model, one simply analyzes additional multiple-sheet models. For example, one could split the conductor thickness into four, eight, or sixteen sheets. This has not been done for this particular line, however experience has shown that almost no change in loss results. When two or more sheets are used to model a transmission line, be certain all the Sonnet analysis ports are numbered as shown in the figure.

Sonnet uses two parameters for loss analysis. R_{DC} is the basic resistive loss at dc in ohms/square for each sheet of conductor. R_{RF} is the skin effect square root of frequency loss. Specifically,

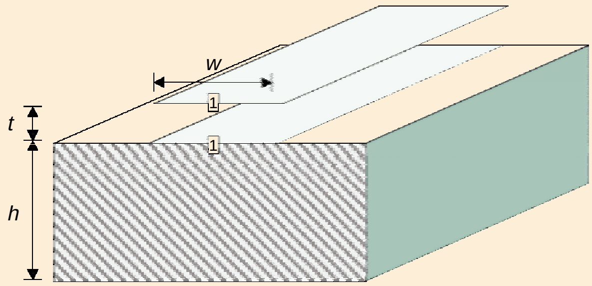
$$R_{DC} = \frac{n}{\sigma t}$$

$$R_{RF} = \sqrt{\frac{\pi\mu}{\sigma}}$$

where n is the number of sheet conductors used in the model.



Typical transmission line to be modeled.



Sonnet two-sheet conductor loss model. Note both sheets at a port location carry the same port number.

L = inductance per unit length ($= Z_0 / v$, v = velocity of propagation(m/s)) (H/m).

This gives a critical frequency of 20 MHz. In addition, this hypothesis gives reasonable answers for all other cases checked. While this equation seems reasonable and gives good results, it is purely empirical and has not been verified by rigorous derivation from Maxwell's equations.

High Frequency Problem

We are still left with an unexplained difference between calculated loss and the square-root model at high frequency (Figure 2). Resistance is increasing faster than the square root of frequency. This suggests that something more than just simple skin effect is impacting high frequency loss.

The solution to this puzzle is found in the fact that microstrip is dispersive. It is dispersive because the dielectric is inhomogeneous. In other words, the fields of the microstrip line encounter more than one kind of dielectric. When a transmission line is dispersive, things like characteristic impedance and velocity of propagation change with frequency. In order for these quantities to change, the current distribution must also change. If the current distribution changes, then the resistance must also change.

Qualitatively, when resistance increases, we expect the current to become, in some way, restricted to a smaller cross-section of the conductor. For example, we might expect the current to become more strongly concentrated on the edge or on the bottom side as frequency increases.

Quantitatively, the calculated resistance increases 22% more than the square-root-of-frequency model over the range of 3 GHz to 20 GHz. The ratio of the current density

The n in the equation for R_{DC}, equals 2 for the two sheet loss model. This is because low frequency current is divided between two parallel connected conductors, each having half the thickness and requiring twice the R_{DC} of a single sheet conductor.

To calculate the total surface impedance, one might be tempted to simply add the low frequency R_{DC} to the high frequency resistivity. Such a model not only yields incorrect results for resistivity, it also ignores the fact that a complex surface impedance (with equal real and imaginary parts) is required at high frequency. Rather, Sonnet calculates the total, frequency-dependent surface impedance (for normal conductors) according to

$$Z_S = \frac{(1+j)R_{\mathrm{RF}}\sqrt{f}}{1-e^{-\frac{(1+j)R_{\mathrm{RF}}\sqrt{f}}{R_{\mathrm{DC}}}}}$$

where Z_S is the total surface impedance (ohms/square). This equation is derived by truncating the usual exponential integral used to calculate skin depth.

At low frequency, $Z_S = R_{DC}$. At high frequency, $Z_S = (1+j)R_{RF}$ multiplied by the square root of frequency. When more than an octave above f_{c2}, only R_{RF} is important. When more than an octave below f_{c2}, only R_{DC} is important. In fact, the square of the ratio of the two sheet R_{DC}/R_{RF} set equal to unity defines f_{c2}. (If a situation is encountered with only one significant sheet of skin depth current flowing, use the one sheet R_{DC}.)

To calculate R_{DC} and R_{RF} for the validation experiment, bulk conductivity was measured with an HP3458A digital multimeter in two-wire mode, calibrated so as to remove probe contact resistance. Four through lines were measured with an average bulk conductivity of 3.42×10^7 S/m. From this, the two-sheet R_{DC} for the 9 µm thick line is 0.0064 ohms/square, and R_{RF} is 3.4×10^{-7}.

A single-conductor Sonnet loss model is also possible and, in fact, is most commonly used. In this case, when calculating R_{DC}, use $n = 1$. For R_{RF}, we must include the effect of two sheets of current in a model that uses a single sheet of current. If the current splits exactly equally between the top and bottom of the conductor, simply use one-half the calculated value of R_{RF}. With microstrip, current concentrates on the bottom side, so a value closer to the above R_{RF} should be used. The equivalent single sheet R_{RF} is given by

$$R'_{\mathrm{RF}} = R_{\mathrm{RF}}(k_1^2 + k_2^2)$$

where

R'_{RF} = equivalent single sheet RRF

R_{RF} = high frequency resistivity parameter for Sonnet loss model

k_1 = fractional top side current

k_2 = Fractional bottom side current ($k_1 + k_2 = 1.0$).

This equation assumes the total transmission-line current is the same in both the one-sheet and two-sheet models. Use of R'_{RF} in the one-sheet model then yields the same power dissipation as R_{RF} in the two-sheet model. The values of k_1 and k_2 can be determined from an initial analysis using the two-sheet model. Use the Sonnet (or Sonnet Lite) visualization program emvu to evaluate the top and bottom current split quantitatively.

Microstrip dispersion causes the value of k_1 and k_2 to vary as a function of frequency. If the values from a single frequency are used to set R'_{RF} for all frequencies, error of up to about 0.1 dB is seen for the transmission line described in this paper.

For two or more conducting sheet models, use R_{RF} unmodified.

flowing in the center on the bottom side of the metal to the top side increases 11%. Thus, at high frequency for the geometry of Figure 1, current does indeed tend to concentrate on the bottom side.

The ratio of the total current flowing in one cell width on the edge to the current in the center of the bottom side increases 15%. Thus, the edge singularity does indeed become stronger at high frequency.

It appears that the increasing edge singularity is, at least in this case, more important than current shifting from the top side to the bottom side, but both effects are important.

Experimental Verification

We can compute all we want, but no research is complete without experimental verification. Measured data is presented for a 6,888-µm- long line on GaAs. Analysis geometry is shown in Figure 1. This line is analyzed by first using Sonnet (or Sonnet Lite) to analyze a line with 1/32 of the total length. This result is then cascaded with itself 32 times, as shown in the Sonnet netlist of Figure 4.

When doing this sort of cascaded analysis, accuracy is critical. Any error in the initial analysis is multiplied 32 times. For this reason, Sonnet's deembedding capability combined with a very small cell size (described earlier) is used. In fact, this sort of cascaded analysis is an excellent way to reveal analysis error when validating software.

Figure 5 shows the measured versus calculated results for this through line. Measured data was taken on an HP 8510C vector automated network analyzer. A SOLT calibration was performed, with the deembedding reference planes set to the ground-signal-ground (GSG) probe tips. Cascade Microtech 150-µm GSG probes were used.

Data was measured for four separate through lines. S_{12} and S_{21} magnitude from all four lines (for a total of eight values) were averaged and plotted. In addition, twice the (unbiased) sample standard deviation (two sigma) is plotted at the top of the graph. The standard deviation gives an indication of the sum of fabrication variation plus measurement repeatability. Absolute measurement error is uncharacterized.

In a tradition that I hope will become established in high frequency publication, I shall refrain from describing the agreement between measured and calculated as "good," which is a subjective conclusion properly left to the informed reader. As the author, I consider it proper only to point out that the calculated data is nearly everywhere within two sigma of the measured data and often within one sigma.

The calculated data is based entirely on physical measurements and dc resistance measurements made independently of the high frequency measurements. There has been no attempt to "tune" resistance or physical parameters to achieve a better fit.

Figure 5 shows that the high frequency range (greater than 363 MHz) varies according to the Sonnet loss model to within measurement error. As can be seen in Figure 2, for most of this frequency range. loss is not square root of frequency.

Only a few data points are within the medium frequency region, 20 MHz to 363 MHz. However the f_{c2} transition region also influences data above the 363 MHz critical frequency. While a rigorous validation requires more data in the medium frequency range, the data presented suggests the medium frequency transition is correctly modeled to within measurement error. The especially small measurement error over this frequency range further strengthens the medium frequency loss- model validation.

Measured data is unavailable below 50 MHz, and thus the low frequency transition frequency, f_{c1}, remains unconfirmed by measurement. This would be a useful measurement, awaiting access to accurate low frequency measurement capability.

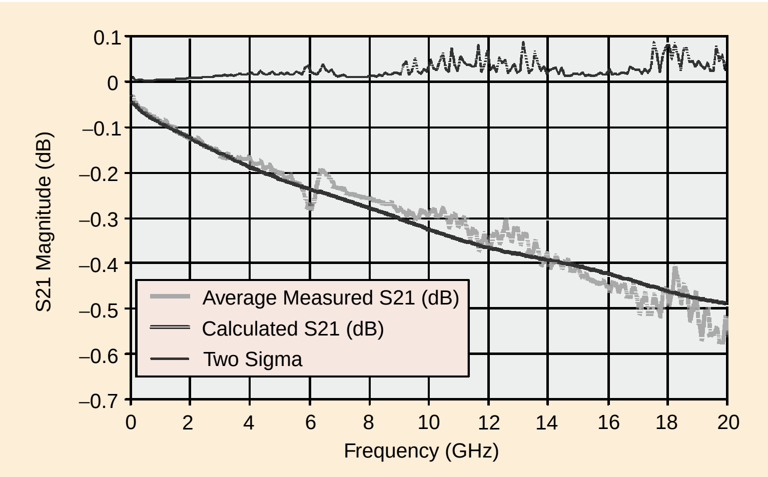


Figure 5. *Results for the 6,888-µm-long line shows differences between average measured data and calculated data are nearly everywhere less than two sigma and often less than one sigma. (Sigma is the sample standard deviation, sample size = 8.)*

These results were obtained without consideration of surface or edge roughness. Roughness can be important if the vertical scale (i.e., average vertical peak-to-valley distance) is larger than or on the order of skin depth (1). In addition, the horizontal scale (i.e., average horizontal distance from one peak to the next) must be on the order of skin depth. If the horizontal scale is much larger or smaller than the skin depth, the impact of surface roughness is reduced.

These results assume a perfectly conducting ground plane. The actual ground-plane conductivity was not measured. The current distribution in the ground plane is essentially uniform under the line, lacking the loss-generating edge singularity seen in the transmission line itself. In this case, simulation indicates the inclusion of finite ground-plane conductivity should add a few percent to the total high frequency loss.

While Figure 5 might once more tempt us to conclude we finally understand loss, the critical observer will point out that inclusion of surface roughness and ground-plane loss might shift the calculated data outside the two-sigma range of the measured data. For further research, quantification of the absolute measurement error, measurement of the actual ground plane bulk conductivity, and visual inspection of the surface and edge roughness would be appropriate.

Implications of the Loss Model

One might think that the low and medium transition frequencies are so low that they are unimportant in applied design. Often, this is true. However, there are exceptions with which a careful high frequency designer should be familiar.

In the case considered, f_{c2} is 363 MHz for a 9-µm-thick line on GaAs. However, this frequency varies inversely with the square of the thickness. Thus, a 1-µm-thick line has f_{c2} over 29 GHz! Now, the entire measured frequency range of the line falls in the medium frequency range.

The first critical frequency is important in resistor design. Resistors properly operate only in the low frequency range, below f_{c1}. If a planar resistor is operated near or above f_{c1}, the resistance is higher than the design value. Such situations are not common, but they do exist. For example, a resistor 100-µm wide, using metal 0.1-µm thick, with a bulk conductivity of 1.0×10^7 S/m, has a resistivity of 1.0 Ω per square and f_{c1} of 6.4 GHz. Depending on design tolerances, the designer might want to use caution above about 3 GHz for this resistor configuration.

It is possible for the order of the critical frequencies to be reversed. The condition for this reversal, which is independent of bulk conductivity, is

$$\frac{t}{w} > 8\frac{L}{\mu} \tag{4}$$

where w is the conductor width.

This situation is most likely seen in low-impedance, ultra-thin dielectric situations. For example, a 5-Ω line with an effective dielectric constant of 3, a line width of 8 µm, and a line thickness of 2 µm has f_{c1} = 10 GHz and f_{c2} = 7.3 GHz.

Sometimes a transmission line has a width that is much smaller than its thickness. In this case, most of the current flows on the sides of the line, rather than on the top and bottom. This loss model is still valid, provided that thickness is taken as the smaller of the two dimensions. If thickness and width are about equal, significant current flows on all four sides of the line and a more sophisticated four conductor sheet model can be used. Alternatively, an equivalent one or two sheet model can be developed, using the equivalent resistance technique described in the sidebar.

Finally, the most significant result impacting applied high frequency design is that microstrip loss increases faster than the square root of frequency above f_{c2}. This is due to microstrip dispersion, where the edge singularity becomes stronger and current concentrates on the underside of the line as frequency increases. This result also offers a possible explanation of a common high frequency designer's complaint, "The measured loss is higher than predicted." Optimistic predictions are presumably based on a simple square-root-of-frequency loss model.

Conclusion

Microstrip conductor loss exhibits complicated behavior that is not generally recognized. Specifically, there are three frequency ranges of interest. At low frequency, current is uniform through the entire cross-section of the line, and the line behaves like a resistor. At medium frequency, the edge singularity forms. In this case, current concentrates on the edge of the line, increasing the resistance. At high frequency, the current splits into two sheets of current, one on top of the line, the other on the bottom of the line. Since microstrip dispersion causes the edge singularity to become larger and current to concentrate on the bottom side as frequency increases, the total resistance increases faster than the normally expected square root of frequency.

Acknowledgments

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