A De-Embedding Algorithm for Electromagnetics

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ABSTRACT

A "double delay" de-embedding algorithm appropriate for electromagnetic analyses is described. This algorithm uses only two standards, a through and a double length through. By evaluating these standards, a special class of port discontinuities may be characterized and removed from the data calculated for a complete structure. Unlike related physical de-embedding algorithms, both the characteristic impedance and the velocity of propagation of the through lines are determined. The technique described here is difficult to implement in a physical de-embedding. The de-embedding theory also provides a new definition of characteristic impedance, "equivalent TEM impedance," for inhomogeneous media, such as microstrip. *This new impedance exhibits a nonmonotonic dispersion which has been* measured experimentally but is not seen using previous impedance definitions.

INTRODUCTION

Electromagnetics researchers are giving increasing attention to the subject of de-embedding. Until now, simply solving a problem and implementing the solution in a computer code was adequate. Results are now needed which can be used in a microwave design environment. De-embedding techniques can provide these results.

Any electromagnetic analysis must have sources (excitations) to excite the structure under analysis. The physical analog is the coax-to-microstrip connector in a test fixture (Fig. 1). Unfortunately, these electromagnetic sources are all too real. Just like physical coaxial connectors, electromagnetic sources also introduce a discontinuity into the result. These discontinuities are due to the evanescent, reactive, fringing fields surrounding the source. Their contribution to the calculated input impedance must be removed if accurate results are to be obtained. In many cases, the reference plane must also be shifted from the source to the device under test (DUT). The process of doing this is called de-embedding and has been well developed over the last several decades for microwave measurements. This field is just now receiving attention in electromagnetics.

We first provide a background on de-embedding algorithms in both microwave measurements and electromagnetics. Next, the detailed theory of a de-embedding algorithm tailored specifically for electromagnetics is described. The algorithm was first developed in October 1987, and has been used extensively in a commercially available electromagnetic program [3]. Finally, a side result of the de-embedding algorithm, a new definition of characteristic impedance suitable for inhomogeneous media, is suggested.

BACKGROUND

Slotted line techniques were first developed in the early days of microwave measurements. A probe which moves along a slot in a waveguide is used to measure the electric field. By comparing the observed standing wave relative to the same measurement for a short circuit at the reference plane,

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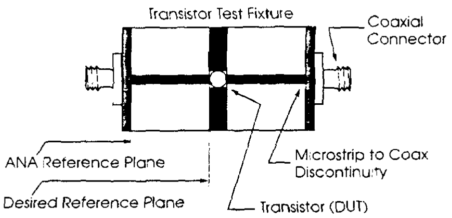

Figure 1. Electromagnetic analyses and physical measurements have port discontinuities in common.

the input impedance (normalized to the waveguide impedance) of a device under test is determined. Because slotted line techniques are now rarely used for physical measurements, we refer to them as "ancient" de-embedding theory.

Modern de-embedding theory, on the other hand, makes use of measurements of a set of known standards; i.e., "short-open-load" or "through-reflect-line." From the measurement of these standards, the port discontinuity (and any intervening lengths of transmission line) can be characterized and then removed from subsequent measurements. These measurements are normalized to the impedance of a nonreflecting (usually 50-Ω) load, or to the characteristic impedance of a (usually 50-Ω) through line standard.

Historically, S-parameters normalized to the characteristic impedance of the connecting transmission lines are used in electromagnetics. This is the kind of S-parameter provided by slotted line techniques and thus slotted line techniques have found considerable use in electromagnetics. The analyses use a connecting transmission line which is long enough to allow a standing wave. The standing wave is usually determined by looking at the current distribution (e.g., on microstrip). Results are easily compared with other electromagnetic results, since they all usually use these same non–50-Ω S-parameters.

When results are required in a practical microwave design environment, the S-parameters must be converted to 50 Ω. The principle difficulty lies in determining the characteristic impedance to which the non–50-Ω S-parameters are normalized. Any error in this determination translates directly into error in the resulting 50-Ω S-parameters. The problem is especially acute for inhomogeneous media, such as microstrip, where no unique definition of characteristic impedance exists.

An additional problem with using slotted line techniques in electromagnetics is that each port must have a long enough transmission line to allow fringing fields to die out and a standing wave to be established. These long transmission lines, which must all be included in the same analysis, can stress an already numerically intensive calculation.

The first published effort applying modern de-embedding theory to electromagnetics [1] requires the evaluation of the "square root" of the cascading matrix of a through line. The difficulty with this approach is described in the next section. A technique which appears to be similar to that described here was also developed more recently [2]. Our application of modern de-embedding theory to the electromagnetic analysis software, *em*, is described next [3,4].

THE DE-EMBEDDING ALGORITHM

The problem to be solved is illustrated in Figure 2. Data for the device under test is modified by the port discontinuities, illustrated here as simple shunt capacitors (the reason for this is described later). The object of the de-embedding is to characterize the port discontinuities. Once we have the S-parameters of the discontinuity, we convert the S-parameters to ABCD (cascading) parameters, invert the ABCD matrix, and pre- and post-multiply the ABCD matrix of the device under test (DUT) with the inverted ABCD matrix of the port discontinuity. This removes the port discontinuity, leaving only the de-embedded S-parameters of the device under test.

The two standards used to characterize the port discontinuity are illustrated in Figure 3. Each standard is a through line with the same geometry as the line connecting the device under test to the port being de-embedded, thus each through line has the same port discontinuity, C. The impedance of the through lines need not be known. We can determine the ABCD matrix of the port discontinuity cascaded with itself as follows:

1. Calculate the ABCD (cascading) matrix for both L and $2L$.

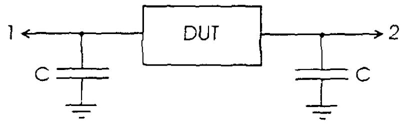

Figure 2. The port discontinuities (C) surrounding the device under test (DUT) must be removed from the electromagnetic result.

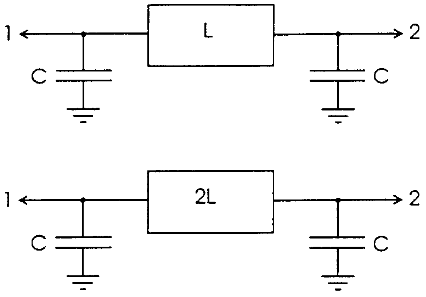

Figure 3. The two standards required for the electromagnetic de-embedding are a through of length L and a through of length $2L$. L needs to be only large enough that the fringing fields from the port discontinuities do not interact.

2. Invert the $2L$ ABCD matrix. This gives us the ABCD matrix for a negative $2L$ length transmission line with inverted port discontinuities on each port.
3. Pre- and post-multiply the inverted $2L$ ABCD matrix with the L ABCD matrix. This leaves the ABCD matrix of the port discontinuity cascaded with itself. We call this result the "double port discontinuity" ABCD matrix.

Until this point, we have made no specializing assumptions about the port discontinuity. We shall now assume that the port discontinuity is, as illustrated, a shunt impedance (e.g., capacitance).

That we cannot simply take the square root of the ABCD matrix of a cascade of two identical, unspecialized port discontinuities is evident from the following observations. An unspecialized (i.e., completely arbitrary) port discontinuity could be modeled as containing a transformer. Let us take the case when the port discontinuity is only a transformer and this port discontinuity is embedding a transmission line (Fig. 4). We have calculated (or measured) the ABCD matrix of the embedded transmission line. From Figure 4, we can pick any value of N and obtain the same embedded ABCD matrix. Thus, from external data (calculated or measured outside unspecialized port discontinuities), it is impossible to uniquely determine the port discontinuity, or for that matter, the impedance of any embedded transmission line. Incidentally, this is why, with few exceptions [7], measurements of characteristic impedance are rare. It is difficult to specialize physical port discontinuities.

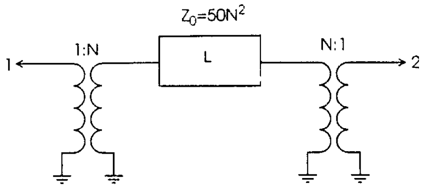

Figure 4. An unspecialized and uncharacterized port discontinuity might transform the impedance level to any unknown value, as illustrated conceptually with the transformers.

We have specialized the port discontinuity to a shunt impedance, Z_S. To verify whether or not this assumption is valid for a particular analysis is simple. The ABCD matrix of a shunt impedance is: A = 1.0, B = 0.0, C = $1.0/Z_S$, and D = 1.0. If A, B, or D of the double port discontinuity ABCD matrix are different from the stated values, then this specialization is not valid for the particular electromagnetic analysis (or measurement). Since the longitudinal dimension of the source (one-half of a subsection) is much less than a wavelength, the source discontinuity is lumped and, with no series impedance, cannot act as a transformer.

Typically, if the port discontinuity has electrical length (is not lumped) or has any tranverse current (series inductance), then this specialization is not valid. This is the case in ref. 1 and for unshielded analyses in general. This is also the case for physical measurements, which is why application of this technique to physical measurements is difficult. In our case (Fig. 5), the source has short electrical length (one-half subsection) and generates no transverse current. Critical in allowing this specialization is the fact that the analysis is of a circuit contained in a conducting box. The perfectly conducting sidewalls provide a perfect

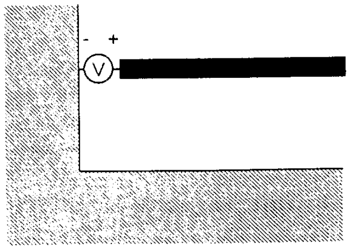

Figure 5. The source (side view) in the *em* analysis goes directly to a grounding sidewall. The same voltage is applied across the entire width of the line, thus there is no transverse electric field and no transverse current.

ground and allow the source to be of lumped dimensions. There is no need to integrate electric field through a substrate to establish a circuit voltage. The evaluation of the double port discontinuity ABCD matrix by *em* shows the port specialization to be valid, for *em*, to within the numerical precision of the computer.

To evaluate the port discontinuity by itself, provided A, B, and D are correct, simply divide C by 2. Now, with the port discontinuity characterized, we can remove the port discontinuity from the device under test by appropriate pre- and post-multiplication of the inverted port discontinuity ABCD matrix. If the second port is a different transmission line (with a different port discontinuity), the de-embedding procedure must be repeated using appropriate through lines.

The reference plane may be shifted, if desired, by length L by multiplying the ABCD matrix of the device under test by the inverted ABCD matrix of the de-embedded L length through.

This algorithm is extended to N-coupled ports by treating each of the four elements of the ABCD matrix as an $N \times N$ matrix. Now, two N-coupled through line standards are needed. Only one de-embedding per box side is needed, no matter how many ports there are. Extension of slotted line techniques to coupled ports is conceptually possible but has never been done. We refer to this algorithm as "double delay" de-embedding, or more simply, as "D-squared" or "D^2" de-embedding.

EVALUATION OF THROUGH LINE PARAMETERS

With the de-embedding complete, we can obtain the transmission line characteristic impedance and effective velocity of propagation. The ABCD matrix for an ideal transmission line is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & jZ_0 \sin(\beta l) \\ j\dfrac{\sin(\beta l)}{Z_0} & \cos(\beta l) \end{bmatrix}$$

where βl is the electrical length of the line (radians) and Z_0 is the characteristic impedance of the line. The ABCD matrix of the de-embedded through line is determined as described in the previous section. That result may be used with the above equation to solve for the transmission line parameters (Z_0, $\cos(\beta l)$, and $\sin(\beta l)$). If L is a multiple of a half wavelength, these parameters are not available, however, the de-embedding is still valid. The effect of dispersion is determined by evaluating these parameters as a function of frequency.

Of special interest is the resulting characteristic impedance. The above equation represents the ABCD parameters of an ideal TEM transmission line. The Z_0 which results from setting the TEM ABCD matrix equal to the de-embedded through line ABCD matrix is what we call the "equivalent TEM impedance."

Note that we make no reference to the fields internal to the through line (only to the fields at the lumped-dimensioned ports at the ends of the line) and that we make no reference to the usual two-dimensional definitions of characteristic impedance. The various two-dimensional definitions provide nonunique results when applied to inhomogeneous media, such as microstrip [5], forcing us to decide which is "best." While an explicit criterion for "best" has never been published, an implicit criterion seems to be: *The best characteristic impedance is that impedance, which, when substituted into the equations for the TEM transmission line, provides S-parameters which are as close as possible to their actual values.*

What we have done is turned that objective into the definition of characteristic impedance. Because evaluation of the characteristic impedance according to this definition requires a three-dimensional (in terms of fields) electromagnetic analysis, it can be viewed as a three-dimensional definition of characteristic impedance. This definition is fully valid and unique for all inhomogeneous transmission line media. Further, for homogeneous media, it is equivalent to the two-dimensional impedance definitions.

The equivalent TEM impedance of a 0.635-mm (25 mil) wide line on 0.635-mm thick Alumina is plotted in Figure 6. Results using the various two-dimensional definitions are also given. The decrease, followed by an increase, in the TEM equivalent impedance is consistent with experimental measurements of microstrip impedance [7]. This behavior is not seen when using previous definitions of impedance [5]. The cause of this behavior is not known.

The analysis used a 2.0-mm through length with the box sidewalls 1.7 mm from the edge of the microstrip line. The line was subsectioned into cells 0.026 mm on a side. Complete analysis time (including the underlying electromagnetic analy-

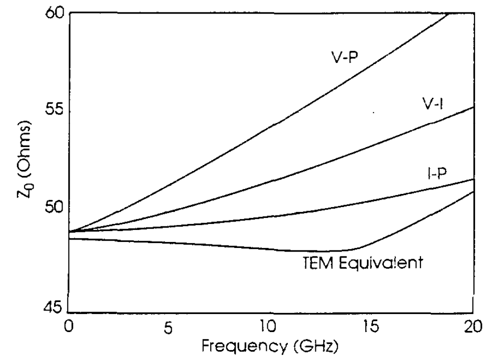

Figure 6. Plot of TEM equivalent characteristic impedance for a 0.635-mm wide line on 0.635-mm thick $\epsilon_{rel} = 9.7$ substrate. Also plotted are three of the usual 2-D impedances (from ref. 5): voltage-power, voltage-current, and current-power.

sis) was 3 minutes per frequency on a SPARCstation 1.

Figure 7 shows the TEM equivalent effective dielectric constant. Describing the dielectric constant as "TEM equivalent" only indicates how the value was obtained. It is identical to the effective dielectric constant as we presently understand it. Also plotted, for reference, is the result of closed form approximations from ref. 6.

Since de-embedded coupled lines can also be evaluated, we could, for example, determine even and odd mode impedances of symmetric couple lines. Other impedances exist for asymmetric coupled lines or multiple coupled lines. Extension of this impedance concept for N-coupled lines is certainly possible, but have not been explored.

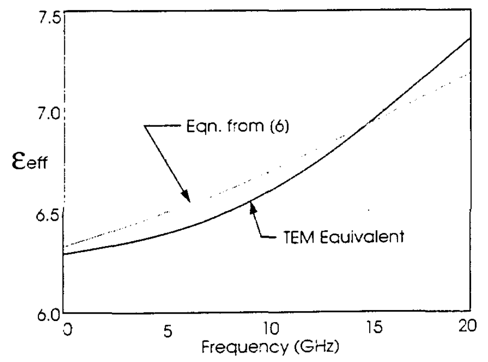

Figure 7. The TEM equivalent effective dielectric constant for the same line as Figure 6. Also plotted for reference is the closed form approximation from ref. 6.

POTENTIAL DE-EMBEDDING DIFFICULTIES

This de-embedding algorithm is based on circuit theory (it can be viewed as interfacing electromagnetic results to circuit theory). As such, there are certain conditions required for the de-embedding to work.

For example, the connecting transmission line may have only one propagating mode. Multimode operation invalidates the results, as is true with physical de-embedding techniques. A common realization of this problem is grounded coplanar waveguide. Both microstrip and coplanar (as well as the usual slot line) modes can propagate. If the ground plane is close enough, significant energy can couple into the microstrip mode and invalidate the results. This is a problem that often exists in on-wafer measurements.

A related restriction is that the port discontinuity can not interact, via fringing fields, with any other discontinuity, such as the device under test or the second port of the through line standard. For example, on 100-μm GaAs, 100 μm is a sufficient separation [8]. This is a restriction that is often violated in physical on-wafer measurements [8], invalidating the results.

Slotted line techniques require that the fringing fields die out and then that there be enough addional length for a measurable standing wave to be established. In addition, the slotted line must be analyzed in the same analysis as the structure of interest, limiting the complexity of the structure to be analyzed. With the double delay technique, de-embedding is performed, means of two separate small analyses.

Package resonances also invalidate results at or close to the resonant frequency.

CONCLUSION

We described a de-embedding algorithm appropriate for electromagnetic analyses. The technique is difficult to apply in physical de-embedding. A side result of the de-embedding, a definition of characteristic impedance appropriate for inhomogeneous transmission lines has also been suggested.

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BIOGRAPHY

James C. Rautio received a BS in electrical engineering from Cornell University in 1978, an MS in Systems Engineering from University of Pennsylvania in 1982 and a PhD in Electrical Engineering from Syracuse University in 1986. From 1982 to 1986, he worked at General Electric, first at the Valley Forge Space Division and then at the Electronics Laboratory, Syracuse. He designed microwave circuits, including filters for a Landsat receiver, and various GaAs microwave integrated circuits. He also developed microwave and millimeter-wave automated measurement equipment and wrote a large microwave circuit analysis program. From 1986 to 1988 he served as a visiting member of the faculty at Syracuse University and as an adjunct faculty member at Cornell University, teaching microwave and computer courses as well as pursuing electromagnetics research. In 1983 he founded Sonnet Software with significant support from the David Sarnoff Research Center, Hewlett-Packard, General Electric, and Syracuse University. He joined the company full time in 1988. Sonnet Software develops and markets electromagnetic software to the microwave design community. Dr. Rautio is a member of IEEE, Tau Beta Pi, ARFTG, AP, and MTT. He has served as chairman of the Syracuse MTT/AP chapter and as a member of the 1987 MTT Symposium Steering Committee.