Fast 3D Planar Electromagnetic Analysis via Unified-FFT Method

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Abstract—Microwave and mm-wave circuits today are larger, faster, and more integrated than ever. This trend continually increases the requirements and applications of electromagnetic simulation. To satisfy this demand, a Unified-FFT method is developed, implemented, and tested, with promising results. The Unified-FFT method combines the fast Matrix Solve Operations (MSO) of the FFT-enhanced Pre-Corrected FFT (PFFT) method with the fast Matrix Fill Operations (MFO) of the FFT-enhanced Sonnet V13. Together, these operations allow for high-accuracy simulation with reduced size constraints, and overall speed improved by more than an order of magnitude.

Index Terms — CAD, CAE, EM simulation, PFFT, Sonnet, Speed-Enhanced

I. Introduction

Solution of the Maxwell equations for 3D planar circuits embedded in a shielded multilayered medium is equivalent to the solution of the electric field integral equation (EFIE):

$$\iint_S \bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,ds' = \mathbf{E}(\mathbf{r}), \qquad (1)$$

where $\bar{\mathbf{G}}(\mathbf{r},\mathbf{r}')$ is the dyadic Green's function of the shielded layered medium [1], and $\mathbf{J}$ is unknown current on conductors of the circuit S. Each term $G_{ab}(\mathbf{r},\mathbf{r}')$, $a,b = x,y,$ of the layered medium dyadic Green's function in the rectangular enclosure is a sum of four terms,

$$\begin{aligned} G_{ab}(\mathbf{r},\mathbf{r}') = \; & s^1_{ab}\Gamma_{ab}(x-x',y-y') \\ & + s^2_{ab}\Gamma_{ab}(x-x',y+y') \\ & + s^3_{ab}\Gamma_{ab}(x+x',y-y') \\ & + s^4_{ab}\Gamma_{ab}(x+x',y+y') \end{aligned} \qquad (2)$$

where $s^1_{ab} = \pm 1$, $i = 1,\ldots,4$, depend on the choice of PEC/PMC boundary conditions on the enclosure walls [1]. Due to the fact that each term Γ_{ab} exhibits translational invariance over coordinates x and y in the enclosure's cross-section, the impedance matrix equation $\mathbf{Z}\cdot\mathbf{I} = \mathbf{V}$ resulting from the Moment Method discretization of (1) allows for both fast evaluation of matrix $\mathbf{Z}$ elements—matrix-fill operations (MFO)—and evaluation of matrix-vector products $\mathbf{Z}\cdot\mathbf{I}$—matrix-solve-operations (MSO)—using FFT. In previous work, fast MFO and fast MSO were implemented separately in Sonnet [1] and in PFFT [2], creating computational bottlenecks in the MSO for the former and MFO in the latter. In this novel work, the fast FFT-based MFO of Sonnet [1] is unified with the fast FFT-based MSO of the PFFT algorithm [2], as shown in Fig. 1.

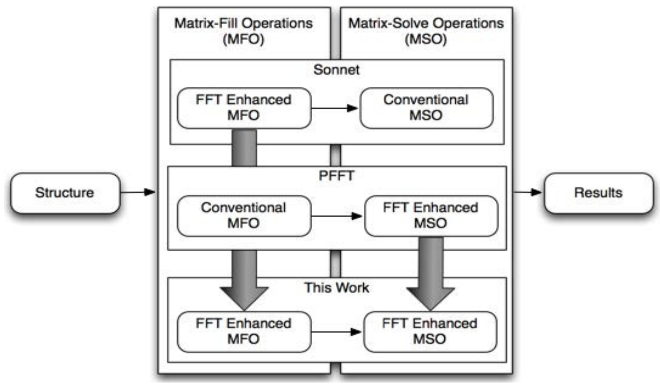

Fig. 1. Block diagram demonstrating Unified-FFT.

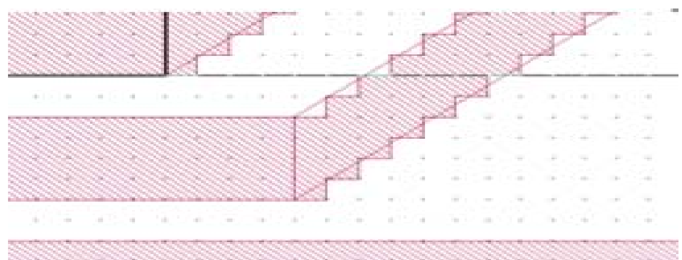

Fig. 2. The uniform FFT grid is represented by a matrix of dots in Sonnet. Vectors "snap" to the grid allowing FFT to be used in MFO.

The computational complexity and memory requirements of the Unified-FFT solver scales a *O(N* log *N)* and *O(N)*, respectively, thus enabling the expedient solution of very dense and electrically large integrated circuits. Accordingly, the Unified-FFT framework is shown to produce fast and accurate full-wave EM analysis of shielded 3D planar circuits of unprecedented sizes.

II. FFT-Enhanced Matrix Fill Operations (MFO)

MFO refers to calculation of matrix $\mathbf{Z}$ elements, which in turn are a relation of voltage on all subsections due to current on one subsection. Ordinarily, this is highly computationally intensive and is shown in [3] to be of the form

$$E_x = \sum_{m=0}^{\infty} \cos\left(\frac{m\pi m_0}{M}\right)\cos\left(\frac{m\pi m_1}{M}\right)$$

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$$* \sum_{n=1}^{\infty} \sin\left(\frac{n\pi n_0}{N}\right) \sin\left(\frac{n\pi n_1}{N}\right) f_{xx}(m,n), \qquad (3)$$

where $f_{xx}(m,n) = G_1(\Delta x) G_2(\Delta y)\left[N_1^2 \hat{Z}_{mn}^{TE} + N_2^2 \hat{Z}_{mn}^{TM}\right]$. One often-noted characteristic of simulation with Sonnet V13 [1] is that geometries are discretized to a uniform grid, as seen in Fig. 2. This uniform distribution is equivalent to uniform spatial-sampling, which allows the use of the FFT to enhance evaluation speed of (3), with its similarity to a Fourier series.

III. FFT-Enhanced Matrix Solve Operations (MSO)

Due to translational invariance of Green's functions components in (2), the scattered E-field integral in the left-hand side of EFIE has the form of 16 terms, each of which is in the form of two-dimensional convolution, correlation, or convolution-correlation [4] over (x , x') and (y , y') coordinates. Choice of the basis functions in the MoM which conform to a regular $K_1 \times K_2$ grid in the enclosure's cross-section naturally as in [1], or through replacement with equivalent point sources [2], casts each of these 16 terms into the form of discrete convolutions and correlations computed using FFT as follows

$$\iint_S \Gamma_{ab}\left(x_{k_1,k_2} \pm x', y_{k_1,k_2} \pm y'\right) J^b(x',y')\,dx'dy'$$
$$= \sum_{k'_1=0}^{K_1-1} \sum_{k'_2=0}^{K_2-1} \Gamma_{ab}\left[(k_1 \pm k_1')\Delta x, (k_2 \pm k_2')\Delta y\right] J^b_{k'_1,k'_2} \qquad (4)$$
$$= FFT^{-1}\left\{FFT\{\Gamma_{ab}\} \times FFT\{J^b\}\right\}, \quad a,b = x,y$$

where $(k_1\Delta x, k_2\Delta y),\ k_1 = 0,\dots,K_1-1;\ k_2 = 0,\dots,K_2-1,$ are samples of the observation locations on the $K_1 \times K_2$ grid for the scattered field, and, $(k'_1\Delta x, k'_2\Delta y)$, $k'_1 = 0,\dots,K_1-1;\ k'_2 = 0,\dots,K_2-1$, are the samples of the source locations for the discretized current.

IV. Engineering The Unified-FFT Solver

While the shielded, 3D-planar nature of Sonnet and PFFT as implemented in this work fundamentally allow integration, the implementation of them together (Unified-FFT) is non-trivial. A robust interface between the two codebases is written in Matlab [5] with the freely-available SonnetLab toolbox [6]. This toolbox allows for vast control of Sonnet projects, with access to Sonnet's default MFO routines with standard Sonnet V13. Notably, Sonnet's MFO are considerably faster in Unified-FFT than in conventional Sonnet. This is because PFFT's MSO routines only require a small portion of the MoM Matrix to be filled, as shown in Section V.

Additionally, Matlab is used to convert geometry and data into a format acceptable for PFFT's MSO routines. Data is transferred via file writes and calls are automatically made to the executable via command line. Moreover, some original 32-bit PFFT code from [2] is modified and recompiled with a modern 64-bit compiler for greatly increased performance and capability, and to further streamline the Unified-FFT package while allowing simulations exceeding 2 GB of memory.

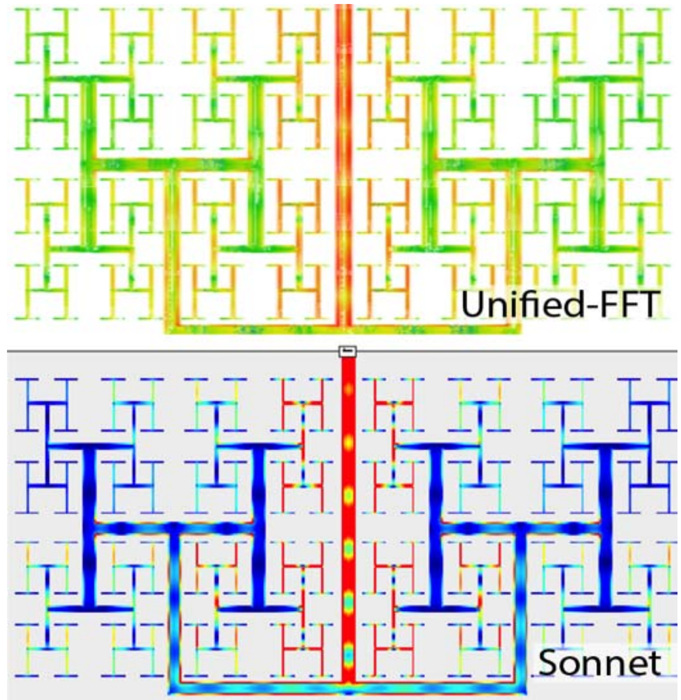

Fig. 3. Current distribution of a 30,000-unknown mock feed-network as simulated in Unified-FFT (above) and Sonnet V13 (below). Differences are due to the color-mapping algorithms by respective software packages.

V. Results and Discussions

Unified-FFT has been implemented and tested on several sample circuits, ranging from $N < 400$ to $N > 50\,000$ unknowns. In general, results deviate from conventional simulation by 1% – 3%. Performance scales very well, with Unified-FFT demonstrating speed-ups well in excess of 10×. It is important to acknowledge that, due to its simulation-focus and prototype nature, this work is comparing Unified-FFT results to Sonnet results, as opposed to measured data. Comparisons are made for both accuracy and performance, and Sonnet is assumed to be a solid baseline due to years of rigorous validation efforts [7].

A. Accuracy

Initial testing is very promising with regards to accuracy, which is validated in three ways. First, most important in a physical sense, is that of current distributions. Fig. 3 shows two current distributions of the same circuit, a mock feed-network consisting of 30,000 unknowns. The upper part of the figure is plotted from results simulated by Unified-FFT, and the lower is directly plotted in Sonnet V13. Aside from color

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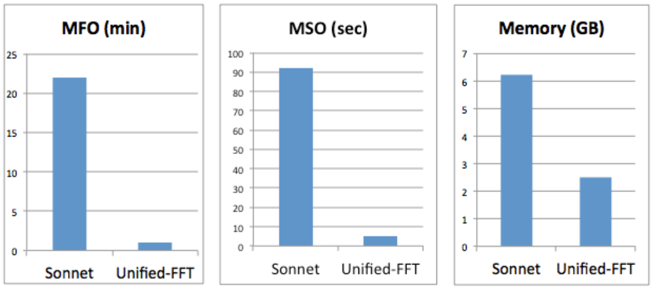

Fig. 4. A comparison between Sonnet and Unified-FFT of MFO and MSO timing, as well as memory requirements for a problem with 30,000 unknowns.

mapping differences due to graphing engines, the distributions are indeed identical.

Second, Y-Parameters (at circuit ports) from the simulations are compared. In this instance, results are generally within 1% – 3% of conventional Sonnet simulation.

Finally, individual matrix elements are compared after MSO is complete. Here, 1% – 3% generally holds, however, in areas with low current levels, error can be as high as 15%. Fortunately, this generally occurs in areas with significantly less contribution to extracted circuit parameters.

B. Performance

Small circuits do not see performance improvements vs. conventional simulation (i.e., Sonnet V13) due to un-amortized overhead incurred during setup of the PFFT algorithm. However, these circuits are sufficiently small such that they already simulate within seconds. As N increases beyond approximately 5 000 – 7 000 unknowns, the $O(N \log N)$ scaling dominates performance. At 10 000 unknowns, MSO is typically 2× faster than conventional simulation, with the MFO being several orders of magnitude faster. By 30 000 unknowns, Unified-FFT becomes an order-of-magnitude faster, as seen in Fig. 4. It is noteworthy that, despite scaling $O(N)$, the memory requirements are relatively higher than the MSO and MFO. This is due to incurred-overheads, and can likely be reduced with further optimizations.

C. An Application-Inspired Example

A mock-up example is created (Fig. 5) for future work, to mimic the types of very large, highly-integrated circuits that are rapidly becoming popular. The circuit is an extended version of the feed-network from Fig. 3, including several spirals. While not a production-type design, it serves well to mimic the type of large circuit functionality being demanded of modern simulators. The circuit features over 50,000 unknowns, which is close to the limit of conventional simulators and thus the practical ceiling for comparative purposes.

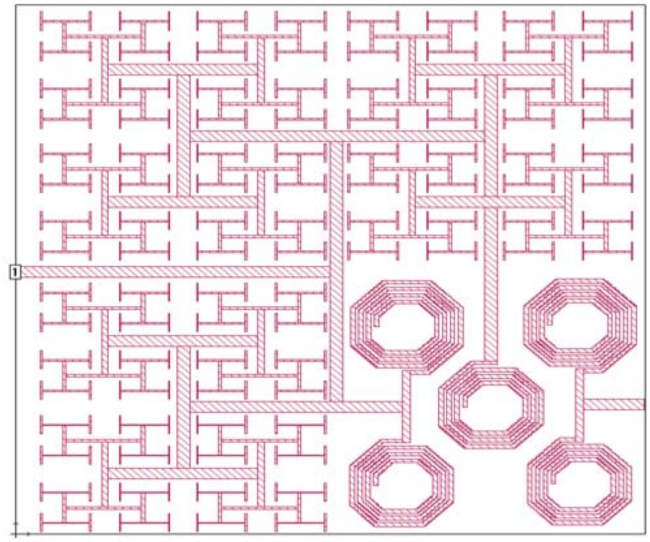

Fig. 5: Another circuit mock-up useful for prototyping the Unified-FFT tool. It intends to capture the essence of modern SoCs/RFICs at a basic level.

Once rigorous characterization of accuracy is established, this circuit as well as considerably more complex circuits will be simulated using UFFT and detailed in future work. Preliminary estimates suggest memory limitations for Unified-FFT will exceed 500 000 unknowns.

VI. Conclusion

The Unified-FFT method has been developed, prototyped, and tested with very positive results, by combining the fast MSO of the FFT-enhanced pre-corrected FFT method with the fast MFO of the FFT-enhanced Sonnet V13.

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