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COMMEMORATIVE FEATURE

Twenty Three Years: The Acceptance of Maxwell's Theory

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by James C. Rautio, Sonnet Software

On October 21, 1805, flying splinters of wood wreak carnage in the battle of Trafalgar as the final and perhaps greatest sailing ship battle of all time is fought to its gory end. Twenty-seven of Lord Nelson's British battle ships with 2150 cannon face Napoleon's fleet of 33 French and Spanish ships carrying 2640 cannon. At the height of battle, Lord Nelson's flag ship, the HMS *Victory*, becomes locked, side-by-side with the French ship *Redoubtable*, their rigging entangled. The British crew gains the upper hand by reducing the gun powder in each charge, allowing the cannon balls to bounce around inside, which wrecks even more carnage against the French ship (see ***Figure 1***).

While Lord Nelson himself does not survive the battle, the British navy is victorious due to the extensive damage inflicted by its radical gun powder tactic and a major storm which soon scatters the remaining survivors. Today you can view a statue of Nelson on a hundred foot column in the exact center of London's Trafalgar Square, signifying the importance of this battle to Britain's history. Little known is the fact that the radical battle-winning strategy Nelson used was first suggested by John Clerk of Eldin, great-great uncle of James Clerk Maxwell.[1]

Fig. 1 Lord Nelson's flag ship, HMS Victory, and Lord Nelson's final moments.

Britain's victory assures British dominance of the oceans for the remainder of the 19th century. As part of this dominance, Britain will discover and secure a monopoly on the production of "gutta percha," a natural plastic made from the sap of a tropical tree. Gutta percha will become the perfect, and for many years, the only practical insulator for the undersea cables used by the early communication networks, which will link the far-flung dominions of the Empire.

Flash forward 60 years to 1865. The American Civil War is ending. Maxwell publishes "A Dynamical Theory of the Electromagnetic Field" in the *Royal Society Transactions*, Vol. CLV (he actually presented the paper orally in December 1864). Comparing several measurements of the speed of light to that calculated by his new electromagnetic theory, he notes, "The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance propagated through the field according to electromagnetic laws."

The directly measured values for the speed of light that Maxwell quotes are 314,858,000 m/s (M. Fizeau), 298,000,000 m/s (M. Foucault) and 308,000,000 m/s (by stellar aberration). Measurements of a capacitor discharge applied to Maxwell's theory yields 310,740,000 m/s (M.M. Weber and Kohlrausch). Given our modern knowledge of the speed of light, we know which results are presented to appropriate precision and with minimum error, and even with that knowledge, Maxwell's conclusion is strong.

Thus, one of the greatest problems of physics is now solved. Or is it? It is actually too early to break out the champagne, for we must patiently wait 23 years. One problem is that Maxwell offers no mechanical model for the "luminiferous ether," the medium in which this supposed wave travels. Maxwell and friends all know that light is a transverse wave (assuming that it is actually a wave, not a particle as Newton had insisted). So whatever medium you propose, it must not allow a longitudinal wave. The medium cannot have translational stiffness. The earth plows through this medium without spiraling into the sun, so either it has no mechanical effect on matter or it has no shear strength. But without shear strength, it can not support a transverse wave. And if there is no interaction with matter, how can any wave get started? And so, Maxwell's theory is just a bunch of equations with no model whatsoever.

And what a bunch of equations—there are 20 of them, simultaneous differential equations, rather than the four equations we know today. Maxwell has the concepts of divergence (he uses the opposite sign and calls it convergence) and curl. But vector calculus is not yet formalized, so he must write out all 20 equations. To do so, he sometimes relies on the use of "quaternions." Quaternions are a combination of a scalar and a vector, which require the use of a squared magnitude of –1, further complicating the equations.

Wait, there's more. Those 20 equations are not easily recognized today. Maxwell places as primary something he calls "electromagnetic momentum" (because its time derivative is force). Electric and magnetic fields are secondary. His friend, Michael Faraday, who originated the field concept as an alternative to the then popular "action at a distance," called it the "electrotonic state." It is, Faraday said, changes in the electrotonic state surrounding magnets that cause magnetic induction. Maxwell formalized Faraday's field concept. The electrotonic state is today called the magnetic vector potential, usually introduced only in graduate level EM courses as a side-effect of a cute little vector identity (primacy of the vector potential is returning to popularity in physics).

Maxwell viewed magnetic vector potential as primary (presumably why he gave it the symbol A) and magnetic field as secondary (presumably why he gave it the symbol B). However, by making the vector (and scalar) potentials primary, Maxwell's equations became complicated. And so, very few took the time to learn them.

Finally, recognition of his work would take longer because of Maxwell himself. Unlike Newton, Maxwell was not an aggressive self-promoter. For example, while president of Section A of the British Association, he gave a presidential address (published in Vol. 2 of the new British journal *Nature*) at the 1870 annual meeting with high praise for a vortex theory of molecules due to his good friend William Thomson (later Lord Kelvin). Rather than wave the flag to the scientific world about his electromagnetic theory, he only mentioned briefly at the end, "Another theory of electricity which I prefer…," not even taking credit for his own work.[2]

Maxwellian: FitzGerald

Maxwell died in 1879. There was no one, no students or colleagues, to carry on his work in electromagnetics. Well, almost no one. Two days after Maxwell's death, the Royal Society mailed a paper review that had been written months earlier by Maxwell back to George Francis FitzGerald (see ***Figure 2***). FitzGerald was a fellow and soon to be professor of Trinity College Dublin.[3] The future of electromagnetics would now lie in the hands of FitzGerald.

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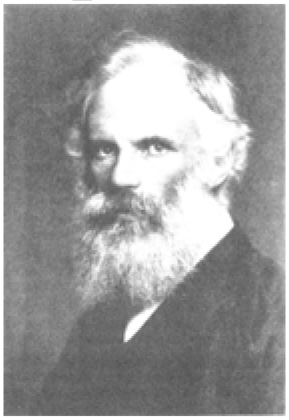

Fig. 2 George Francis FitzGerald, one of the few scientists to pursue Maxwell's theory.

FitzGerald was a brilliant idea man, although self-described as "lazy" when it came to follow up experimental work. Typically, he would do the initial work and then rely on others (usually friends or students) to continue the effort. In the paper that Maxwell reviewed, FitzGerald had linked Maxwell's electrodynamic theory to an earlier theory of Prof. James MacCullagh also of Trinity College. This theory modeled the luminiferous ether required by Fresnel's wave theory of light and required a purely rotational elasticity, i.e., no translational stress was allowed to form. MacCullagh had shown that given this form for the ether, one could model refraction, reflection and polarization perfectly. Quite the coincidence. However, there were a couple of problems. First, MacCullagh did not suggest a physical form for this mysterious ether (it is certainly unknown in normal matter). Second, in 1862, G.G. Stokes, in reviewing a number of proposed ethereal models, pointed out that MacCullagh's ether violates conservation of angular momentum. A nice idea while it lasted.

Maxwell published his treatise on electromagnetics (the founding document of our field) in 1873. However, in this, and in all of Maxwell's EM theory publications, there is no electromagnetic treatment of reflection or refraction. FitzGerald is one of the very few people who read and learned the treatise in detail. FitzGerald also extensively studied MacCullagh's ether model while preparing for his fellowship exam. His well annotated copy of Maxwell's treatise includes a long note dated 7 September 1878 where he first mentions that it might be possible to connect Maxwell's theory to MacCullagh's. Over-simplifying just a bit, FitzGerald found a mapping of variables from Maxwell's theory to MacCullagh's. In his paper he describes the mapping and points out that MacCullagh's work now brings reflection and refraction to Maxwell's theory.

However, Stokes's objections still hold; MacCullagh's ether does not conserve angular momentum. If MacCullagh's ether is the same as Maxwell's, then Maxwell cannot be correct either. FitzGerald, a life long believer in some kind of ether, optimistically announces that perhaps we should "emancipate our minds from the thralldom of a material ether."

Maxwellian: Lodge

Oliver Lodge (see ***Figure 3***) was the son of a pottery clay merchant. His path takes a higher road when he wins a scholarship and completes a University of London external degree. He completes a doctorate at University College in London and later becomes the first professor of physics at the new University College in Liverpool in 1881. Lodge had acquired a copy of Maxwell's treatise in 1873, right after it was published. He started studying it in 1876. He first met FitzGerald at the August 1878 Dublin meeting of the British Association. Sharing a strong interest in Maxwell's treatise, they quickly became fast friends.

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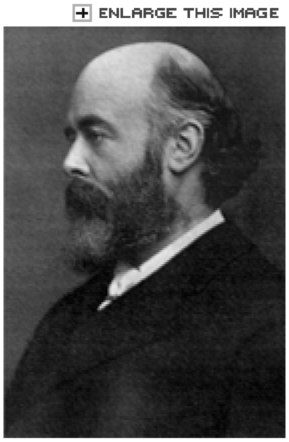

Fig. 3 Oliver Lodge worked with FitzGerald in puzzling out Maxwell's theory.

A great unsolved mystery is why Maxwell never tried to produce electromagnetic waves. Historians suggest that Maxwell himself simply missed the implication of his theory. The possibility that changing currents could make light and that radiation could happen is not obvious from Maxwell's equations, especially in the form Maxwell used. Maxwell appears to have believed that light was made by mechanically vibrating molecules somehow coupling to and vibrating the ether. And EM waves of long wavelength? At that time, such "dark light" was outside human experience.

Lodge was more comfortable working with mechanical models, rather than directly with equations. In 1879, one of his models suggested that it might be possible to create light electromagnetically. But how? Lodge proposed applying a voltage source through a switch that switches ("breaks") 400 times a second. The square wave is applied to a coil that somehow doubles the frequency. A cascade of 40 such coils should yield light. FitzGerald gently points out that the square wave would be smoothed into a sine wave after only several coils and thus would not work.

Lodge, not one to give up easily, suggests that a discharging condenser (i.e., capacitor, in the form of a Leyden jar) is oscillatory. FitzGerald's response was not preserved, but he likely told Lodge that the frequency would be too low for light. So close, yet they missed the idea that the experiment would generate long wavelengths. If only they could find a way to "see" such wavelengths.

FitzGerald then pursued the problem mathematically, using Maxwell's treatise and making several errors along the way. First he reads Maxwell too literally, as though the treatise is an electromagnetic bible. For example, Maxwell states repeatedly that his theory gives results equivalent to the old "action at a distance" concept. In this case, there can be no radiation. However, it appears that Maxwell's comment was limited to the non-time varying situation, something FitzGerald did not realize. A second very serious error concerns what is now called the gauge condition. Maxwell selected what we call the Colomb gauge (divergence of A is zero) and then incorrectly specified the scalar potential to be independent of time, yielding a static solution. Eventually FitzGerald realizes the problem, in part by toying with mechanical models of the ether that he had built. Then likely inspired by Lord Rayleigh's Theory of Sound, and by electromagnetic research published by Lorenz, he introduces retarded potentials to Maxwell's theory.

FitzGerald, with considerable effort, does find a solution to Maxwell's equations for a time varying current, but it is a non-radiating solution, "like the nodes and loops in an organ pipe." He thus concludes that generating EM waves electrically is impossible. What FitzGerald did not realize at the time, is that he had unwittingly found the solution assuming a conducting wall boundary condition. This illustrates a problem of the vector potential; boundary conditions are difficult to see. Years later FitzGerald realizes that an alternative solution (assuming a different boundary condition) takes the form of traveling waves. Regardless, the damage is done. FitzGerald and Lodge continue enthusiastically working with Maxwell's theory, but the search for EM wave generation is terminated... until 1888.

Maxwellian: Heaviside

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Fig. 4 Oliver Heaviside, a reclusive mathematical genius, cast Maxwell's equations into their modern form.

Described by a friend[4] as a "first rate oddity," Oliver Heaviside (see ***Figure 4***) was also an exceptionally prolific writer, albeit difficult to read, and an absolute mathematical genius. He was raised in poverty in Camden Town, London around the corner from where Dickens himself had lived. In addition to the harsh treatment, he was often ill and never attended a university. Beyond a reasonable early education (partially provided by his mother), he was self-taught in science and mathematics. He learned by reading books from the library. It appears he passionately avoided books on theology and metaphysics (unlike Maxwell), but he adored books dealing with Newton and Laplace, to name a few.

His started his first and only job in 1867 as a telegraph clerk. His uncle by marriage, Charles Whetstone (of bridge fame), helped him obtain the position. Because of its importance to the Empire, much of British science was focused on problems relating to undersea telegraph cables. The first undersea cable (Dover to Calais) was laid in 1851. By 1885, there are nearly 100,000 miles of cable under the ocean, mostly laid by Britain due to their dominance on the ocean and monopoly in the only viable undersea cable insulator, gutta percha.

Heaviside obtained a copy of Maxwell's treatise in 1873. "I browsed through it and I was astonished! I read the preface and the last chapter, and several bits here and there; I saw that it was great, greater and greatest… I was determined to master the book and set to work." Heaviside left his job the next year and moved in with his parents to pursue the mathematics of Maxwell, especially with regard to telegraphic (and soon, telephonic) cable problems.

Heaviside had no patience with perceived stupidity and one person in particular, William Henry Preece, the Engineer-in-Chief of the British General Post Office (which controlled all British telegraph and telephone lines). In turn, Preece, who considered himself to be an especially intelligent, "practical man," had no need for theoretical mathematicians.

In one case, Heaviside derived the transmission line "telegraphers" equation directly from Maxwell's theory. William Thomson (Lord Kelvin) had successfully analyzed undersea cables based on the diffusion equation, i.e., just resistance and capacitance, but no inductance. In this case, a pulse effectively diffuses into the cable and Thomson's model provided reasonable results for most undersea cables but failed miserably for overhead lines. When Heaviside derived the full telegrapher's equation, he determined that if the ratio of L/R is equal to C/G, distortion (i.e., pulse spreading) could be eliminated. Since G is very small, thanks to gutta percha, decreasing R to the required value would be prohibitively expensive. So, the solution to addressing distortion was simply increase L. However, his work is effectively suppressed over a long period by Preece, who was vehement that the inductance of a transmission line is zero and increasing it would only lead to disaster. As a "practical man," he could not be convinced otherwise by these silly mathematicians. Later engineers in the United States successfully make, apply and patent the same discovery, to which Heaviside received no credit.

In the summer of 1884, Heaviside starts working on energy flow in the electromagnetic field. The derivation is complicated, but the result is simple: S=ExH. Heaviside, being reclusive and not well connected with the rest of the scientific community, is later only a little disappointed to find that Prof. Poynting of the new Mason College of Science in Birmingham had published the same result a few months earlier.

As an example of energy flow, take the field around a straight copper wire at DC. ExH points radially into the wire. Energy does not flow along the wire as had been thought. It flows from the field surrounding the wire and dissipates as heat as it enters the wire. This is the big clue. In sharp contrast to action at a distance, where energy is viewed as flowing along the wire with the current like water in a pipe, Maxwell's equations suggest that energy is in the field and flows from the field into the resistance of the conductor.

In fact, the Maxwellian view at this time is that charge and current are not physical. Rather they are changes in the stresses and strains of the ether. The conductor of the wire relieves the stress of the field and dissipates the energy as heat. This view fades with the advent of a new concept a few years later, the electron. As for the modern view, electromagnetic energy is calculated in terms of either the field or the current. It's hard to say precisely "where" it is.

Heaviside went further than Poynting. As Heaviside was working with his energy concept, he came upon a new form for Maxwell's equations, the "duplex" form, the four equations with which we are familiar today. These differential curl equations involve E, H, D and B. The potentials are gleefully "murdered" according to Heaviside. "I never made any progress until I threw all the potentials overboard," he later wrote to FitzGerald. With the duplex form, the symmetries in Maxwell's equations are beautifully seen, but something was missing. Heaviside adds the fictitious magnetic current to complete the symmetry.

If Heaviside modified Maxwell's equations to this degree, why don't we call them Heaviside's equations? Heaviside answered this question in the preface to Vol. 1 of his three-volume, lifetime culminating work, *Electromagnetic Theory*, stating that if we have good reason, "to believe that he [Maxwell] would have admitted the necessity of change when pointed out to him, then I think the resulting modified theory may well be called Maxwell's."

In 1888, Lodge is requested to give two lectures on lightning protection and he conducts experiments by discharging condensers in the vicinity of models of various structures. The lectures stimulate major controversy with Preece as Lodge's results contradict standard practice. But all that is minor. In the course of his experiments, Lodge notices arcs being induced in nearby circuits. In one experiment, he sees an arc at the end of two parallel wires. In the dark, he can even see a distinct standing wave glowing in the air around the wires. Lodge has generated and detected electromagnetic waves. The British Association is meeting in Bath in September. In July, Lodge plans to report his astounding results at that meeting, right after he returns from vacation in the Alps. On the train out of Liverpool, he picks up the July issue of *Annalen der Physik*. He immediately notices an article, "Ueber elektrodynamische Wellen in Luftraume und deren Reflexion" ("On Electromagnetic Waves in Air and Their Reflection").

Maxwellian: Hertz

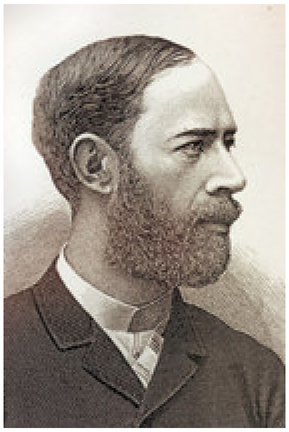

Fig. 5 Heinrich Hertz experimentally validated Maxwell's equations 23 years after Maxwell first published them.

Heinrich Hertz (see ***Figure 5***) studied in Berlin under Hermann von Helmholtz. Helmholtz had incorporated the electromagnetic theories of Maxwell, Weber and Neumann into a single theory with a parameter (k), whose value selected the theory to be used (later it would be realized that Maxwell's theories were not correctly incorporated fully). Helmholtz had encouraged Hertz to perform experiments to test and differentiate the theories. Hertz at first declined, having determined that the experiments would be difficult to perform. However, a few years later, while teaching at Technische Hochschule in Karlsruhe, he noticed while discharging a condenser through a loop, that an identical loop some distance away developed arcs. He instantly recognized a resonance condition and suspected electromagnetic waves. The experiments he subsequently conducted would verify reflection, refraction, diffraction and polarization for both free space waves and wire guided waves. It is Hertz's experimental results that Lodge is reading in *Annalen der Physik* as he leaves for vacation.

As Lodge read the paper, he realized that his own results were now superfluous. However, his disappointment is more than compensated by the beauty and completeness of Hertz's work. Hertz presents at the September 1888 British Association meeting in Bath and is hailed a hero. His results provide full confirmation of Maxwell's electromagnetic theory. The British Maxwellians, after 15 years of careful theoretical preparation of Maxwell's theory, are catapulted to the top of British science thanks to Hertz's timely experimental validation. Hertz is warmly welcomed into the small Maxwellian group and takes an active role, but, unfortunately, for far to short a period of time. In Germany, the importance of Hertz's results is not at first fully recognized. Some will say (only partly in jest), that word of Hertz's experiments reached Germany by way of England. However, once recognized, German researchers likewise embrace Maxwell's theory as well.

Hertz had, independently of Heaviside, discarded Maxwell's potentials and developed the modern duplex form of Maxwell's equations. When Hertz becomes aware of Heaviside's work, he graciously yields priority to Heaviside and likewise chooses to call them Maxwell's equations. As a tribute to Hertz, they are for a few years also sometimes called the Hertz-Maxwell equations.

Epilog

Hertz returned to England for one final visit to receive the prestigious Rumford Medal from the Royal Society in 1890. The year before, Hertz had received a major promotion and moved to the chair of physics at the University of Bonn. Unfortunately, the move also deprived him of experimental facilities. In close collaboration with the British Maxwellians, Hertz continued to make significant theoretical contributions, although work becomes difficult as he starts suffering from an infection that spreads to his jaw and sinuses. He writes to his parents in August 1892, "At present my nose is my universe." He dies tragically and painfully at the age of 36 from blood poisoning after an operation.

Michelson and Morley performed their interferometer experiment in 1887, which casts doubt on the existence of an ether. British researchers continue searching for mechanical models of the ether well into the 20th century before the effort is gradually dropped as pointless.

FitzGerald passes away after a particularly difficult bout of indigestion at the age of 49. To Heaviside and Lodge his death is a great shock. Lodge does not contribute any further work to Maxwell's theory after 1900, shifting his efforts toward researching communication with the dead. He died on August 22, 1940, promising to make public appearances after his death. No such appearances have been recorded.

In 1896, Heaviside's father dies, leaving him on his own for the first time in his life. FitzGerald and John Perry arrange a Civil List pension for him. In a more difficult task, they convince him to accept it, offering it as recognition of service to his country. Heaviside became senile in his old age ("I am as stupid as an owl"). He dies on February 3, 1925 after falling off a ladder and landing on his back. He is 74. His ride to the hospital is his first and final ride in an automobile.

I close with a paragraph written January 30, 1891 in Heaviside's *Electromagnetic Theory*: "Lastly, from millions of vibrations per second, proceed to billions, and we come to light (and heat) radiation, which are, in Maxwell's theory, identified with electromagnetic disturbances. The great gap between Hertzian waves and waves of light has not yet been bridged, but I do not doubt that it will be done by the discovery of improved methods of generating and observing very short waves."

We truly do stand on the shoulders of giants.

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