A Volume Current Based Method of Moments Analysis of Shielded Planar 3-D Circuits in Layered Media

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***Abstract*—Method of moments analysis of planar multi-layer circuits typically assumes conductors are infinitely thin and only surface currents need be modeled. Modern fabrication methods, especially for high frequency integrated circuits, can easily create structures that require modeling volume current. Combined with previous work, this paper presents, for the first time, a complete *volume current* based method of moments analysis of shielded multi-layer circuits. The new volume and the original surface subsections are all evaluated to full numerical precision by 2-D FFT and have little impact on analysis speed.**

***Keywords*— Electromagnetic, Galerkin, integrated circuit, method of moments, planar, RFIC, rooftop, thick conductor.**

I. Introduction

Method of moments [1] as applied to electromagnetic (EM) analysis of planar circuits has been widely implemented for both unshielded, for example, [2] and shielded [3] planar circuits in layered media by assuming 2-D, infinitely thin sheets of current are sufficient to model actual microwave circuits. However, modern circuit fabrication, especially for Si and GaAs microwave integrated circuits, can fabricate planar microwave components with conductors that are thick compared to line widths or line-to-line gaps. A typical example is shown in Fig. 1, from [4].

One solution is to use a multi-sheet model of thickness [5] or any of a variety of volume meshing analyses. However, excessive compute resources are required for complex circuits.

Instead, we model the actual volume current using volume subsections. Use of a surface current 'rooftop' subsection [6] (i.e., basis function), has been widespread. This has been extended to a volume rooftop subsection [7]. Modelling horizontal volume currents using volume rooftop subsections in a multi-layer circuit also requires a 'via' subsection that allows current to flow vertically from one layer to another. As shown in this paper, two types of via subsections are required. First, a via that has uniform current flowing along its vertical length, and second, a via whose current tapers linearly from zero at the bottom to maximum at the top, i.e., a 'tapered via'. Full method of moments reaction integral results are provided for both types of via in this paper.

These results, when combined with results for horizontal volume current rooftop subsections [6] provide for the first time a complete *volume current* based method of moments analysis of multi-layered planar shielded circuits. This method meshes only the conductor in a circuit resulting in a substantially smaller problem compared to meshing the entire space. Analysis time is independent of whether volume or surface mesh is used for a given number of subsections.

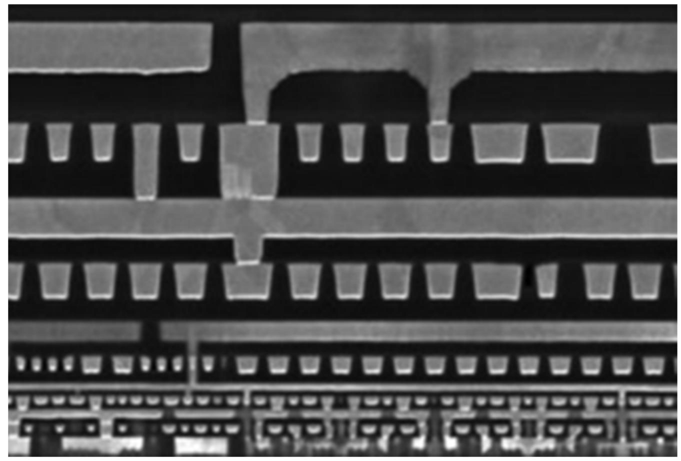

Fig. 1. SEM (Scanning Electron Microscope) image of the cross section of a silicon RF integrated circuit using the Intel's 14 nm process with 52 nm minimum pitch. Line thickness (vertical) can be larger than the line width (horizontal) and larger than the gap between lines as well, creating an extreme EM analysis problem for large circuits. From [4].

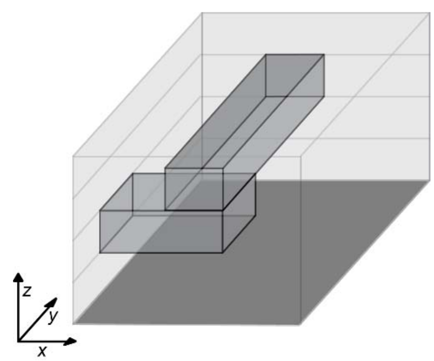

Fig. 2. An illustrative multi-layer shielded planar circuit requires *z*-directed current to make the electrical connection from the right end of the lower conductor to the near end of the upper conductor. Because the conductors are thick, both horizontal and vertical currents are best modelled by volume current basis functions. From [8].

II. Volume Via Types

Fig. 2 shows a simple, illustrative planar circuit using thick conductors in shielded layered media. The portion of the circuit in which only horizontal current flows is accurately modelled using only volume rooftop subsections. However, the junction region where the two conductors overlap requires vertical current.

Consider the region of transition, where the lines overlap and touch. This region consists of two 'blocks'. The bottom block envelops the right end of the lower line. The top block envelops the near end of the upper line. It is in these two regions that vertical (z-directed) current flows. Fig. 3a shows a single block including dimension nomenclature.

Considering a block enveloping the right end of the bottom line in Fig. 2, the vertical current must be zero on the bottom surface. Only horizontal current can flow there. At the top surface of the bottom block (where the bottom and top blocks touch), we have maximum vertical current. This is where all the previously horizontal current from the bottom line is now flowing vertically to connect with the near end of the top line.

Thus, in order to model the vertical current in the bottom block, we need a via subsection with zero current at the bottom and maximum current at the top, i.e., a tapered via [8]. We also refer to this as an 'up via' as it is used to transition current upwards from a horizontal volume rooftop. If not specified, a tapered via is here always assumed to be an up via. Note that the 'taper' refers only to the vertical current density variation within the via. The physical shape of a tapered via is a rectangular prism, i.e., a rectangular block, Fig. 3a.

Consider now the vertical current in the top block, at the near end of the top line. The top surface of the top block must have zero vertical current. All current flow has transitioned to horizontal current flow in the volume rooftops used to model the top line.

The bottom surface of the top block, where it meets the top surface of the bottom block, must have maximum vertical current as all the previously horizontal current from the lower conductor is flowing vertically across this surface.

Thus, we need a tapered via that has zero current at the top and maximum current at the bottom in order to model the vertical component of current in this region. We also refer to this as a 'down via' [8] as it can be used to transition current downwards. In fact, a down via positioned on the next layer up from an up via (as is the situation in this example) actually together form a vertical volume rooftop subsection that extends across two adjacent dielectric layers.

III. Via Relations

Fig. 3a shows a single dimensioned via block. Fig. 3b shows the magnitude of the vertical volume current within a via block when a tapered volume up via subsection (i.e., basis function) occupies that block. The current is the same everywhere in any horizontal cross-section, and it tapers linearly from zero at the bottom to a maximum at the top, or, assuming the base of the block is at $z = 0$,

$$J_z = z/h \tag{1}$$

Fig. 3c illustrates the current distribution on a 'uniform via', i.e., a via that has constant current along its entire length.

Fig. 3d shows the total vertical current in a block that is occupied by both an up via and a uniform via. If arbitrary and independent magnitudes, or weights, can be assigned to each of the two co-located vias (as is the usual approach in method of moments), then we can model current with any slope at any level.

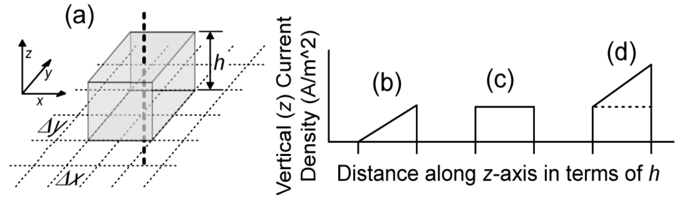

Fig. 3. A single, dimensioned, via block (a) and the variation in z-directed current along the z-direction for an up-tapered via (b), a uniform via (c), and for a block that contains both a tapered and uniform via (d). Current for all vias is constant across a given x-y cross-section.

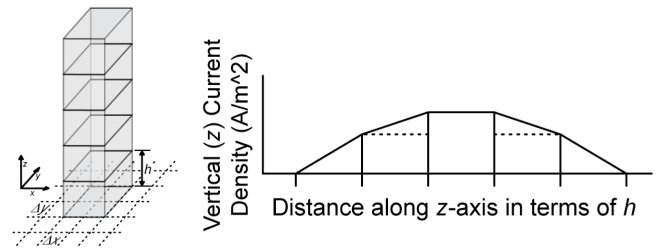

Fig. 4. A lengthy (compared to wavelength) via can be accurately modelled by a stack of vias. The density of the z-directed current along the z-direction is shown on the right. The bottom block needs only an up-tapered via. The top block needs a down-tapered via. All other blocks should be filled with both a tapered and a uniform via.

Such a model is illustrated in Fig. 4 for a via stack, yielding a piecewise linear representation of the current on the via stack. The bottom end of the stack contains only an up via. The top end needs only a down via. All other blocks contain both an up via and a uniform via.

An actual down via is not required. If a block is occupied by both an up via and a uniform via, method of moments assigns a negative weight to the up via when a down via is required. This subtracts the up via current from the uniform via current creating the effect of a down via. Fundamentally, a down via is an up via subtracted from a uniform via. Thus, of the three types of vias: down, uniform, and up; only two should be used in any given volume block as the third is a linear combination of the other two. Use of all three via types collocated within the same block results in a singular moment matrix. Here, we consider only the uniform via and up via. If a down via is needed it is formed by subtracting an up via from a uniform via.

Note that a down via placed directly above an up via effectively forms a vertically oriented volume rooftop basis function that crosses over two dielectric layers. This, combined with the horizontal x- and y-directed volume rooftops are the key basis functions required for modelling completely arbitrary 3-D structures embedded in shielded multi-layered media.

IV. Via Reaction Integrals

Derivation of the reaction integrals required for method of moments starts by derivation of the full dyadic Green's function in terms of waveguide modes, where the waveguide modes are defined for each dielectric layer by the shielding box sidewalls [4]. The Green's function for horizontal (x-y) fields due to horizontal current is given in [7]. Space precludes presentation of the Green's function due to vertical current here.

Next, the Green's function is multiplied by the tapered via current density and integrated over the volume of the tapered via. This determines the fields that surround the tapered via.

Finally, with the tapered via as the source subsection, a field subsection is selected. The fields of the source subsection are multiplied by the current distribution/basis function of the field subsection and integrated over the volume of the field subsection (Galerkin technique). This yields the desired method of moments reaction integrals.

Using standard waveguide theory nomenclature, where the waveguide is formed by the shielding sidewalls with dimensions a and b and TE and TM mode numbers m and n, we start with modal normalizing constants (2), modal transverse spatial variation (3) – (5), and various required Fourier coefficient related expressions (6) – (8).

$$N_1 = \sqrt{2/ab}\,,\ \ m = 0,\ n > 0 \tag{2a}$$
$$N_1 = 2k_y/k_c\sqrt{ab},\ \ m > 0,\ n > 0 \tag{2b}$$
$$N_2 = \sqrt{2/ab}\,,\ \ m > 0,\ n = 0 \tag{2c}$$
$$N_2 = 2k_x/k_c\sqrt{ab},\ \ m > 0,\ n > 0 \tag{2d}$$
$$N_3 = 2k_c/{k_z}^2\sqrt{ab},\ \ m > 0,\ n > 0 \tag{2e}$$

$$g_1(x,y) = \cos(k_x x)\sin(k_y y) \tag{3}$$
$$g_2(x,y) = \sin(k_x x)\cos(k_y y) \tag{4}$$
$$g_3(x,y) = \sin(k_x x)\sin(k_y y) \tag{5}$$

$$C_{iVIA}(x,y) = F_R(\Delta x)F_R(\Delta y)N_3 g_3(x,y) \tag{6}$$
$$C_{iRFX}(x,y) = F_T(\Delta x)F_R(\Delta y)\,[\boldsymbol{e}_{it}(x,y)\cdot\boldsymbol{u}_x] \tag{7}$$
$$C_{iRFY}(x,y) = F_R(\Delta x)F_T(\Delta y)\,[\boldsymbol{e}_{it}(x,y)\cdot\boldsymbol{u}_y] \tag{8}$$

The F_R and F_T are the Fourier coefficients for a rectangle and triangle pulse respectively given in [7], and the $\boldsymbol{e}_{it}$ are normalized transverse waveguide modes given in [3]. The index i stands in for all TE and TM *m*, *n* modes and for brevity is not always explicit.

A term that often appears in our results is

$$D = (1 - r_{iT}r_{iB})\sin(k_{iz}h) + (r_{iT}+r_{iB})\cos(k_{iz}h) \tag{9}$$

where the r_{iT} are the normalized impedances of the top cover (usually a perfect conductor) transformed down to the top of the via block through the cascade of waveguides formed by each intervening dielectric layer and the shield sidewalls. Likewise, the r_{iB} are the normalized impedances of the bottom cover transformed up to the bottom of the via block.

The desired method of moments reaction integrals of a tapered via to a surface current *x*-directed rooftop (located at the bottom of the dielectric layer), to a volume current *x*-directed rooftop, to a uniform via, and to another tapered via are

$$S_{TVtoRFX} = \sum \frac{C_{VIA}(x_0,y_0)C_{RFX}(x_1,y_1)}{Y_i k_{iz} h}\left(1 - \frac{g_2}{D}\right) \tag{10}$$
$$g_2 = \sin(k_{iz}h) + r_{iT}\cos(k_{iz}h) + r_{iB}(1 - r_{iT}k_{iz}h) \tag{11}$$

$$S_{TVtoVRFX} = \sum \frac{C_{VIA}(x_0,y_0)C_{RFX}(x_1,y_1)}{Y_i k_{iz}^2 h}\left(k_{iz}h - \frac{g_3}{D}\right) \tag{12}$$
$$g_3 = (2 - r_{iT}k_{iz}h)(1 - \cos(k_{iz}h)) + (r_{iT} + r_{iB} - r_{iT}r_{iB}k_{iz}h)\sin(k_{iz}h) \tag{13}$$

$$S_{TVtoUV} = \sum \frac{C_{VIA}(x_0,y_0)C_{VIA}(x_1,y_1)}{Y_i}\left(\frac{g_4}{D} + \frac{k^2}{2k_c^2}k_{iz}h\right) \tag{14}$$
$$g_4 = \left(\frac{r_{iT}-r_{iB}}{k_{iz}h} + r_{iT}r_{iB}\right)(1 - \cos(k_{iz}h)) - r_{iT}\sin(k_{iz}h) \tag{15}$$

$$S_{TVtoTV} = \sum \frac{C_{VIA}(x_0,y_0)C_{VIA}(x_1,y_1)}{Y_i}\left(\frac{g_5}{D} + \frac{k^2}{3k_c^2}k_{iz}h\right) \tag{16}$$
$$\begin{aligned} g_5 = \frac{1}{k_{iz}^2h^2}[&(r_{iT} + r_{iB} - r_{iT}r_{iB}k_{iz}h)k_{iz}h + \\ &\{(r_{iT}r_{iB} + 1)k_{iz}h - r_{iT}k_{iz}^2h^2 - r_{iT} - r_{iB}\}\sin(k_{iz}h) + \\ &\{2 - (r_{iT} - r_{iB})k_{iz}h - r_{iT}r_{iB}k_{iz}^2h^2\}(\cos(k_{iz}h) - 1)]. \end{aligned} \tag{17}$$

Additional reaction integrals for uniform vias are

$$S_{UVtoRFX} = \sum \frac{C_{VIA}(x_0,y_0)C_{RFX}(x_1,y_1)}{Y_i}\left(\frac{g_6}{D}\right) \tag{18}$$
$$g_6 = -r_{iB}\sin(k_{iz}h) - r_{iT}r_{iB}(\cos(k_{iz}h) - 1) \tag{19}$$

$$S_{UVtoVRFX} = \sum \frac{C_{VIA}(x_0,y_0)C_{RFX}(x_1,y_1)}{Y_i k_{iz}}\left(\frac{g_7}{D}\right) \tag{20}$$
$$g_7 = (r_{iT} - r_{iB})(1 - \cos(k_{iz}h)) \tag{21}$$

$$S_{UVtoUV} = \sum \frac{C_{VIA}(x_0,y_0)C_{VIA}(x_1,y_1)}{Y_i}\left(\frac{g_8}{D} + \frac{k^2}{k_c^2}k_{iz}h\right) \tag{22}$$
$$g_8 = 2r_{iT}r_{iB}(1 - \cos(k_{iz}h)) - (r_{iT} + r_{iB})\sin(k_{iz}h). \tag{23}$$

Since only *z*-directed current is involved, all TE mode magnitudes are zero. The modal admittance Y_i, is the standing wave admittance and thus differs from the usual definition by a factor of *j*. Reactions to a *y*-directed rooftop basis function are realized by substituting $C_{RFY}(x_1,y_1)$ in place of $C_{RFX}(x_1,y_1)$.

Combined with the volume rooftop reaction integrals given in [7], this forms for the first time a complete *volume current* based method of moments analysis of 3-D structures embedded in multi-layered media.

This technique meshes only the embedded structure, where the structure is considered everything that is different from the embedding layered media. The layered media is not meshed, nevertheless, its effect is included to full numerical precision within the reaction integrals listed above. In practice, the embedding media can have hundreds, or even over 1000 layers with extreme ranges of thicknesses and constitutive parameters with little impact on analysis speed. For a wide range of problems, this can result in a considerably faster analysis as compared to the various volume meshing techniques.

V. VALIDATION

Since the horizontal volume rooftop subsections have already been carefully validated [7], we select a horizontally oriented coupled line bend as the benchmark, Fig. 5 (left). All dielectric layers are set to free space. The geometry is such that we can rotate the circuit 90° and have the identical physical geometry, Fig. 5 (right). The rotated geometry requires analysis of two via stacks, each similar to the stack in Fig. 4. The figures are to scale, with box size 290 microns cubical, and with lines having a 10 micron square cross-section with a 10 micron gap. Ports 1 and 2 are on the left and the other two ends are shorted to the adjacent sidewall or cover.

Fig. 6 shows S_{21} magnitude (dB) for both circuits. Correct tapered and uniform via theory is required in order for the response of these two circuits to be identical, and indeed, we do see that in Fig. 6. The agreement is so good that the two curves appear exactly one on top of the other.

VI. Conclusion

Combined with the volume rooftop reaction integrals given in [7], this forms for the first time a complete *volume current* based method of moments analysis of 3-D structures embedded in multi-layered media. This analysis approach promises to be useful for design of planar circuits that use thick conductors, a situation now becoming common for silicon RF integrated circuits. In addition, the volume rooftops combined with the tapered and uniform via basis functions form the key required elements for analysis of completely arbitrary 3-D structures embedded in shielded layered media. The method of moments reaction integrals for both surface and volume meshings used here are evaluated to full numerical precision using a 2-D FFT and typically have little or no impact on analysis time.

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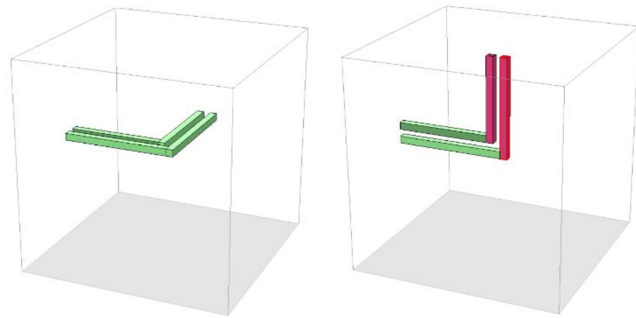

Fig. 5. The coupled line bend on the right is modelled using the previously validated horizontal volume rooftop subsections. The geometry is selected so that it can be rotated 90° and the circuit is physically identical. However the vertical portion (red) of the rotated circuit (right) must be analysed using a superposition of both tapered and uniform vias in each via block.

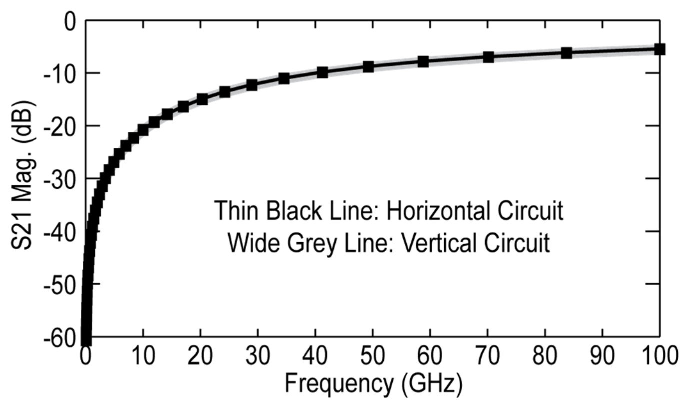

Fig. 6. A plot of S_{21} magnitude (dB) for both circuits of Fig. 5 shows identical results (the two curves exactly overlap), thus validating both the tapered and uniform via theory.