Techniques for Correcting Scattering Parameter Data of an Imperfectly Terminated Multiport When Measured with a Two-Port Network Analyzer

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Abstract—Two techniques are described which correct scattering parameter data taken on an N-port device measured with N-2 imperfect terminations and a two-port network analyzer. The first technique uses a simple iterative algorithm and may be easily implemented in software. Each iteration reduces the error due to imperfect terminations typically by one decade. The second, more complicated, technique uses a general closed-form solution which requires specially developed Gamma-R parameters of which S-, Y-, and Z-parameters are particular cases. The closed-form solution is completely valid for any termination. The closed-form solution is the limit to which the iterative solution converges.

The iterative technique has been implemented in software controlling an HP 8409 automated microwave network analyzer.

I. Introduction

When measurements are required of three-port S-parameters, the usual technique is to terminate each port, in turn, with a reflectionless load and make three separate two-port measurements. The configuration for the first measurement is shown in Fig. 1. The two-port automated network analyzer (ANA) may be considered nearly perfect if one of several correction algorithms [1]–[4] is used. All nine required three-port S-parameters (with some redundancy) may then be selected from the three two-port measurements. The technique may be extended to N-ports with $N(N-1)/2$ two-port measurements required. When imperfect loads are used, the resulting two-port S-parameter data will be corrupted. The error introduced can be removed by a closed-form technique which has been applied to the "Thru-Short-Delay" technique [3], [4]. A rigorous closed-form technique which uses S-parameters normalized to a complex impedance has been described by Tippet and Speciale [24]. Their solution will remove errors introduced by imperfect terminations. However, they state that their technique is limited in the number of perfectly reflecting terminations which may be used.

In this paper, an iterative solution and a new simple closed-form solution will be described. The iterative approach has the advantage of being extremely simple and easily programmed. The closed-form solution, while more elegant than the iterative solution, is less flexible and requires some matrix inversion. Fewer complex matrix inversions and multiplications are required than in [24]. The solution introduces Gamma-R parameters which are similar to S-parameters normalized to an arbitrary impedance [6]–[12] except that Gamma-R parameters do not degenerate for short or open circuits. Thus, the closed-form solution maintains full validity at and in the vicinity of high reflection terminations.

The techniques described in this paper will not remove measurement errors introduced by imperfections in the two-port ANA itself.

II. The Iterative Solution

If the true N-port data are known, the resulting measured two-port data are easily calculated [14], [17]–[20]. For example,

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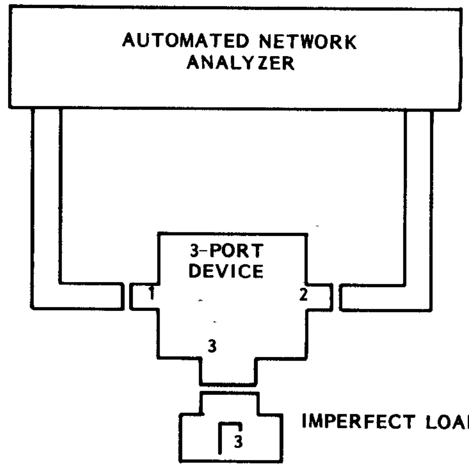

Fig. 1 Measurement setup to obtain the first four of nine three-port S-parameters. The measured data will be corrupted by the imperfect load.

```
      SUBROUTINE ECOR7(SM, GAM, SC)
C*************************************************************************
C                  8409 ANA SUBROUTINE ECOR7
C    This subroutine corrects the measured 3-port data in SM for the
C    effect of the imperfect termination, GAM   The corrected data is
C    returned in SC   SM and SC must occupy physically distinct areas
C    in memory
C    WRITTEN BY J. RAUTIO          9 APR 82           REVISION A-1
C*************************************************************************
C
      COMPLEX*8  SM(9),       ' Measured 3-port data (column major order)
     &           SC(9),       ' Corrected 3-port data
     &           SE(9),       ' Current estimate of true 3-port data
     &           GAM          ' Reflection coefficient of termination
      INTEGER*2  M(9,3),      ' S-parameter indices for calculating error,
                              ' dependent on how device was measured
     &           I, IS        ' Do loop indices
C
C     INDEX  --  1    2    3    4    5    6    7    8    9
C     S-PARAM -- S11  S21  S31  S12  S22  S23  S13  S23  S33
      DATA M/    7,   8,   6,   7,   8,   3,   4,   2,   6,   ' Sit
     &           3,   3,   2,   6,   6,   4,   8,   7,   8,   ' Stj
     &           9,   9,   5,   9,   9,   1,   5,   1,   5/   ' Stt
C
C     Use the measured data as the first estimate of the true data
      DO 100 IS=1,9
        SE(IS)=SM(IS)
 100    CONTINUE
C
C     Using SE as an estimate of the true data, subtract the calculated
C     effect of the load from the measured data and place the improved
C     estimate of the data in SC   Do this six times
      DO 200 I=1,6
C       Go thru each S-parameter
        DO 210 IS=1,9
          SC(IS)= SM(IS) - SE(M(IS,1))*SE(M(IS,2))*GAM/
     &                          (CMPLX(1.,0 )-SE(M(IS,3))*GAM)
 210      CONTINUE
C       Use SC as the new estimate of the true S-parameters
        DO 220 IS=1,9
          SE(IS)=SC(IS)
 220      CONTINUE
 200    CONTINUE
      RETURN
      END
```

Fig. 2 Excluding comment, common, data, etc., statements; the iterative algorithm may be implemented with seven lines of Fortran.

given a three-port with port 3 terminated in a load (reflection coefficient = Γ_3) the resulting two-port S_{11} will be

$$^2S_{11} = {}^3S_{11} + \frac{^3S_{13}\,^3S_{31}\Gamma_3}{1 - {}^3S_{33}\Gamma_3}$$

or, more generally

$$^2S_{ij} = {}^3S_{ij} + \frac{^3S_{it}\,^3S_{tj}\Gamma_t}{1 - {}^3S_{tt}\Gamma_t}$$

where S_{ij} is the S-parameter being measured, and t is the terminated port. The left-hand side of the above equations is the two-port S-parameter measured by the ANA. The first term of the right-hand side is the true three-port S-parameter. The second term is the error caused by the imperfect load.

Solving for the first term of the right-hand side

$$ {}^3S_{ij} = {}^2S_{ij} - \frac{{}^3S_{it}\,{}^3S_{tj}\Gamma_t}{1 - {}^3S_{tt}\Gamma_t}. \tag{1} $$

This equation (or actually, nine equations, one for each of the three-port S-parameters) gives the true three-port data as a function of the measured two-port data, the measured imperfect load, and the unknown three-port data. This is a set of nine simultaneous equations in nine variables. A simple iterative technique may be used to solve the equations. The unknown three-port data in the right-hand side error term may be estimated by the measured two-port data. Equation (1) is then evaluated to yield an improved estimate of the true three-port data. This improved estimate (of all nine S-parameters) is then used to re-evaluate (1) to yield an even better estimate of the three-port data. Typical results show the error (rms difference between the old and new estimate) decreases by one decade for each iteration. The Fortran source for this error-correction algorithm is shown in Fig. 2. Ignoring common, continue, data, and comment statements, the algorithm uses seven lines.

III. Extension to Four-Ports

The equation corresponding to (1) for four-ports may be found several ways [17]–[20]; the technique described by Otoshi [17] is

$$ {}^4S_{ij} = {}^2S_{ij} - \frac{\Gamma_t \begin{vmatrix} S_{it}S_{tj} & -S_{it}S_{tr}\Gamma_r \\ S_{rj} & 1 - S_{rr}\Gamma_r \end{vmatrix} + \Gamma_r \begin{vmatrix} S_{tj} & 1 - S_{tt}\Gamma_t \\ -S_{ir}S_{rj} & S_{ir}S_{rt}\Gamma_t \end{vmatrix}}{\begin{vmatrix} 1 - S_{tt}\Gamma_t & -S_{tr}\Gamma_r \\ -S_{rt}\Gamma_t & 1 - S_{rr}\Gamma_r \end{vmatrix}} $$

where S_{ij} is the S-parameter being measured, t is the first terminated port (1–4), and r is the second terminated port (1–4). The vertical bars indicate determinants, and all S-parameters in the determinants are four-port S-parameters.

The same iterative algorithm can then be used. The extension to N-ports is straight forward. Measurement of more than four ports may become cumbersome.

IV. Extension to Short Circuits

Short circuits, rather than loads, may be used when it is realized that while a reflectionless load will allow accurate measurement of three-port S-parameters on a two-port ANA, a perfect short will allow measurement of three-port Y-parameters on a two-port Y-parameter ANA. While a Y-parameter ANA does not exist, it is simple to convert the measured S-parameters to Y-parameters [5], [15], [21]

$$ Y = \frac{1}{Z_0}\frac{1-S}{1+S} $$

where S is the $N \times N$ S-parameter matrix, Y is the $N \times N$ Y-parameter matrix, and Z_0 is the system impedance (usually 50 Ω). It is pointed out in [5] and [15] that

$$ (1-A)(1+A)^{-1} = (1+A)^{-1}(1-A) $$

for any A, thus the form above.

Conversion back to S-parameters is

$$ S = \frac{1 - Z_0 Y}{1 + Z_0 Y}. $$

Computation time will be saved if one uses normalized Y-parameters ($Z_0 = 1$) for all calculations.

The same iterative correction algorithm may be used with Y-parameters if one substitutes Y-parameters for S-parameters and $-Z$ (input impedance of the short) for Γ_L.

The measurement algorithm for shorts may be summarized as follows.

1) To measure an N-port, first measure N-2 short circuits, calculate minus the input impedance, and store in memory.

2) Measure the S-parameters of the required two-ports using the N-2 shorts to terminate unused ports.

3) Convert the two-port S-parameters to Y-parameters.

4) Correct the measured Y-parameters for the effect of imperfect shorts using the iterative algorithm.

5) Convert the resulting corrected N-port Y-parameters to S-parameters.

In certain situations, this algorithm yields unacceptable errors which are thought to be due to difficulty in measuring the high reflection termination. A similar technique using open circuits and Z-parameters would also be possible but has not been explored.

V. The Closed-Form Solution

As noted above, one may measure the S-parameters of an N-port using a two-port S-parameter ANA and N-2 reflectionless loads. Similarly, one may use N-2 perfect short circuits to measure Y-parameters on a Y-parameter ANA or N-2 perfect opens and a Z-parameter ANA. We can create a Y-parameter ANA by measuring S-parameters and converting the S-parameters to Y-parameters. The same is true for Z-parameters. In a similar manner, it is possible to do this for any arbitrary termination using Gamma-R parameters.

The Gamma-R parameters (derived in Appendix I) use a linear combination of the incident and reflected waves as terminal variables. The combination, which may be different for different ports, is chosen such that the appropriate terminal variable will go to zero when the termination (with any fixed reflection coefficient) associated with the port is attached. A special case would be the reflected wave going to zero when a reflectionless load is attached or the terminal voltage going to zero when a perfect short is attached.

The algorithm for measuring N-ports will be the following.

1) Measure N terminations. Each termination will be associated with a specific port.

2) Measure $N(N\text{-}1)/2$ two-ports. Create each two-port by connecting N-2 of the N terminations to the N-port. Be sure each termination used is connected to its associated port.

3) Convert each of the two-port S-parameters to Gamma-R parameters as follows:

$$ R = (\Gamma^* + S)(1 - \Gamma S)^{-1}. $$

Definitions are in Appendix I and the derivation is in Appendix II. The Gamma matrix will be the $N \times N$ Gamma matrix with all rows and columns corresponding to the terminated ports eliminated.

4) Fill the $N \times N$ Gamma-R matrix with the Gamma-R parameters measured from the two-ports.

5) Convert the final $N \times N$ Gamma-R matrix back to S-parameters as follows:

$$ S = (1 + R\Gamma)^{-1}(R - \Gamma^*). $$

Definitions are in Appendix I and the derivation is in Appendix II.

Use of similar normalized S-parameters for ANA accuracy enhancement has been discussed [13], [22], [23], [24].

VI. Relative Merits

The closed-form solution can allow for any termination reflection coefficient on any port. The iterative method requires something close to a load. However, the iterative solution can handle the terminations connected in any sequence to any of the ports, as long as the exact sequence is known to the correction algorithm. The closed-form solution requires that each port be terminated by the same termination each time it is terminated. It is not possible to terminate, say, port 3 with one termination and later terminate port 3 with a different termination when using the closed-form solution. This is not a difficulty when measuring three-ports because each port is terminated once. Only one physical termination is required. However, a four-port requires four physically distinct terminations to meet the above requirements. In contrast, the iterative technique requires only two physically distinct terminations.

VII. Some Properties of the Gamma-R Parameters

By noting that the port current $i = a - b$ and the port voltage $v = a + b$, we can see (2) that for $\Gamma = -1$ (short circuit), $\boldsymbol{\alpha} = \boldsymbol{v}$ and $\boldsymbol{\beta} = -\boldsymbol{i}$. Thus, the Gamma-$R$ parameters become normalized ($Z_0 = 1$) Y-parameters with a change of sign. Also, if $\Gamma = 1$ (open circuit), we have $\boldsymbol{\alpha} = \boldsymbol{i}$, $\boldsymbol{\beta} = \boldsymbol{v}$ and normalized Z-parameters result. For $\Gamma = 0$, S-parameters result. Thus, S, Y, and Z are all special cases of Gamma-R parameters. Note that the conversion equations (4) and (5) then simplify to conversions between S, $-Y$, and Z.

For a 2×2 Gamma-R matrix, it may be easily shown that if $\Gamma_1 = 0$ and $\Gamma_2 = \Gamma_L$ that R_{11} will be the resulting S_{11} of the two-port terminated in Γ_L.

Finally, for $\Gamma_1 = \Gamma_G$ (generator) and $\Gamma_2 = \Gamma_L$ (load)

$$|R_{21}|^2 = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|(1-S_{11}\Gamma_G)(1-S_{22}\Gamma_L) - S_{12}S_{21}\Gamma_G\Gamma_L|^2}$$

or the power transferred from the generator to the load [16].

VIII. Conclusion

A simple iterative technique has been described which corrects the scattering parameter data of an imperfectly terminated multiport device measured using a two-port ANA. The iterative algorithm is fast, flexible, and quickly implemented in software.

Alternatively, a new closed-form solution may be used to correct the measurements. This solution requires fewer complex matrix inversions than a previous closed-form solution [24] and is completely valid for perfectly reflecting terminations. The new closed-form solution introduces Gamma-R parameters.

Except for measurements using arbitrary (e.g., high reflection) terminations, the iterative solution appears to be more desirable due to its simplicity.

Appendix I
Derivation of the Generalized Gamma-R Parameters

We wish to derive an $N \times N$ matrix R which will describe an N-port using $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as terminal variables

$$\boldsymbol{\alpha}^T = (\alpha_1, \alpha_2, \cdots, \alpha_N) \quad \boldsymbol{\beta}^T = (\beta_1, \beta_2, \cdots, \beta_N), \quad \boldsymbol{\beta} = R\boldsymbol{\alpha}$$
$$\boldsymbol{a}^T = (a_1, a_2, \cdots, a_N) \quad \boldsymbol{b}^T = (b_1, b_2, \cdots, b_N), \quad \boldsymbol{b} = S\boldsymbol{a}.$$

We will choose α and β to be linear combinations of a and b, the incident and reflected waves. The linear (and generally complex) combination will be defined as

$$\boldsymbol{\alpha} = A\boldsymbol{a} + B\boldsymbol{b}, \quad \text{e.g., } \alpha_i = A_i a_i + B_i b_i$$
$$\boldsymbol{\beta} = C\boldsymbol{a} + D\boldsymbol{b}, \quad \text{e.g., } \beta_i = C_i a_i + D_i b_i$$

where A, B, C, and D are diagonal matrices of complex constants

$$A = \begin{pmatrix} A_1 & & & 0 \\ & A_2 & & \\ & & \ddots & \\ 0 & & & A_N \end{pmatrix} \quad \text{etc.}$$

We will outline the gamma matrix as

$$\Gamma = \begin{pmatrix} \Gamma_1 & & & 0 \\ & \Gamma_2 & & \\ & & \ddots & \\ 0 & & & \Gamma_N \end{pmatrix}.$$

We wish to select A, B, C, and D such that α_i will go to zero when Γ_i is used to terminate port i.

Further, we will specify that

$$1)\ (A_i^*, B_i^*)\begin{pmatrix} A_i \\ B_i \end{pmatrix} = 1 + |\Gamma_i|^2, \qquad 1 \leqslant i \leqslant N$$

$$2)\ (C_i^*, D_i^*)\begin{pmatrix} C_i \\ D_i \end{pmatrix} = 1 + |\Gamma_i|^2, \qquad 1 \leqslant i \leqslant N$$

$$3)\ (A_i^*, B_i^*)\begin{pmatrix} C_i \\ D_i \end{pmatrix} = 0, \qquad 1 \leqslant i \leqslant N.$$

Conditions 1 and 2 are a convenient normalization. Condition 3 is an orthogonality condition which results in maximum information in the R matrix. At the other extreme, if $A_i = C_i$ and $B_i = D_i$, then the R matrix would be a unit matrix regardless of the N-port being described. This is, in fact, the situation if $\Gamma = -1$ when using S-parameters normalized to a complex (0) impedance [6]–[12].

When the ith port is terminated, we have

$$a_i = \Gamma_i b_i.$$

Under this circumstance, we want

$$\alpha_i = A_i a_i + B_i b_i = 0$$
$$= A_i \Gamma_i b_i + B_i b_i = 0$$
$$A_i \Gamma_i = -B_i.$$

One solution satisfying the first normality condition 1 is

$$A_i = 1 \quad B_i = -\Gamma_i.$$

By the second normality condition 2 and the orthogonality condition 3

$$C_i = \Gamma_i^* \quad D_i = 1.$$

Since this is true for any port i we have

$$A = 1 \qquad B = -\Gamma$$
$$C = \Gamma^* \qquad D = 1$$

and

$$\boldsymbol{\alpha} = \boldsymbol{a} - \Gamma\boldsymbol{b} \quad \boldsymbol{\beta} = \Gamma^*_{a+b}. \tag{2}$$

Appendix II
Conversion Between Gamma-R and S-Parameters

The Gamma-R parameters and S-parameters are defined as

$$\boldsymbol{\beta} = R\boldsymbol{\alpha} \quad \boldsymbol{b} = S\boldsymbol{a}$$

with $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ defined in (2) as

$$(\Gamma^*\boldsymbol{a} + \boldsymbol{b}) = R(\boldsymbol{a} - \Gamma\boldsymbol{b})$$
$$(\Gamma^*\boldsymbol{a} + S\boldsymbol{a}) = R(\boldsymbol{a} - \Gamma S\boldsymbol{a}) \tag{3}$$
$$(\Gamma^* + S) = R(1 - \Gamma S)$$
$$(\Gamma^* + S)(1 - \Gamma S)^{-1} = R. \tag{4}$$

To convert from Gamma-R to S, we start with (3) and proceed

$$(\Gamma^* + S) = (R - R\Gamma S)$$
$$R - \Gamma^* = S + R\Gamma S$$
$$R - \Gamma^* = (1 + R\Gamma)S$$
$$(1 + R\Gamma)^{-1}(R - \Gamma^*) = S. \tag{5}$$

If all Γ_i are equal, the matrix multiplication in (4) and (5) is cummutative.

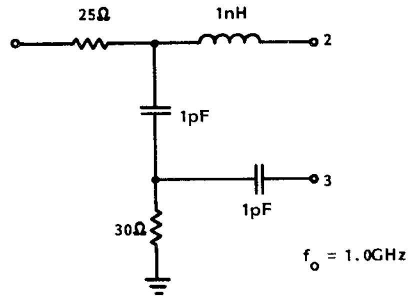

Fig. 3. This circuit is used to demonstrate both the iterative and closed-form solutions.

Matrix M2 was calculated with port 3 terminated in $40 + j10\,\Omega$ ($\Gamma_3 = -0.0976 + j0.1220$) with port 1 as input and port 2 as output. Matrix M3 used a $70 + j0$-Ω load on port 2 with port 1 as input ($\Gamma_2 = 0.1667 + j0.0$) and port 3 as output. Matrix M4 had port 1 terminated with $60 + j10\,\Omega$ ($\Gamma_1 = 0.0984 + j0.0820$) with port 2 as input and port 3 as output.

The three "measurements" were combined to form the "measured" three-port matrix. When possible, the worst data was selected

$$\begin{pmatrix} (0.2739, -0.0994) & (0.7540, -0.1735) & (-0.0328, \ 0.0335) \\ (0.7540, -0.1735) & (0.1878, -0.1294) & (-0.0423, \ 0.0456) \\ (-0.0328, \ 0.0335) & (-0.0423, \ 0.0456) & (0.7636, -0.4974) \end{pmatrix}. \tag{M5}$$

The first iteration provided

$$\begin{pmatrix} (0.1823, -0.0524) & (0.7537, -0.1738) & (-0.0288, \ 0.0262) \\ (0.7537, -0.1738) & (0.1110, -0.1495) & (-0.0377, \ 0.0441) \\ (-0.0288, \ 0.0262) & (-0.0377, \ 0.0441) & (0.7637, -0.4967) \end{pmatrix}. \tag{M6}$$

The second iteration gave

$$\begin{pmatrix} (0.1838, -0.0526) & (0.7538, -0.1737) & (-0.0294, \ 0.0266) \\ (0.7538, -0.1737) & (0.1120, -0.1489) & (-0.0385, \ 0.0446) \\ (-0.0294, \ 0.0266) & (-0.0385, \ 0.0446) & (0.7637, -0.4968) \end{pmatrix}. \tag{M7}$$

Appendix III
Numerical Results

This section will provide specific numerical examples of both the iterative and closed-form solution. These examples will facilitate the verification of software using these techniques.

The circuit in Fig. 3 was analyzed with the assistance of a computer. The true S-parameter matrix at 1 GHz is

$$\begin{pmatrix} (0.1837, -0.0527) & (0.7538, -0.1737) & (-0.0293, \ 0.0265) \\ (0.7538, -0.1737) & (0.1120, -0.1490) & (-0.0384, \ 0.0446) \\ (-0.0293, \ 0.0265) & (-0.0384, \ 0.0446) & (0.7637, -0.4968) \end{pmatrix}. \tag{M1}$$

To demonstrate the iterative technique, each port was terminated, in turn, by an imperfect load. The resulting calculated S-parameter matrices correspond to ANA measurements. The resulting three "measured" two-ports are

$$\begin{pmatrix} (0.1839, -0.0525) & (0.7540, -0.1735) \\ (0.7540, -0.1735) & (0.1124, -0.1486) \end{pmatrix} \tag{M2}$$

$$\begin{pmatrix} (0.2739, -0.0994) & (-0.0328, \ 0.0335) \\ (-0.0328, \ 0.0335) & (0.7636, -0.4974) \end{pmatrix} \tag{M3}$$

$$\begin{pmatrix} (0.1878, -0.1294) & (-0.0423, \ 0.0456) \\ (-0.0423, \ 0.0456) & (0.7639, -0.4969) \end{pmatrix}. \tag{M4}$$

The third iteration result was identical to the original matrix, M1.

The circuit of Fig. 1 was also used to illustrate the closed-form solution. The terminations were changed to $\Gamma_1 = 0.6\angle 35°$, $\Gamma_2 = +1$, $\Gamma_3 = -1$. The "measured" data then becomes

$$\begin{pmatrix} (0.1834, -0.0519) & (0.7535, -0.1725) \\ (0.7535, -0.1725) & (0.1117, -0.1471) \end{pmatrix} \tag{M8}$$

$$\begin{pmatrix} (0.7249, -0.4383) & (-0.0451, \ 0.0746) \\ (-0.0451, \ 0.0745) & (0.7625, -0.5004) \end{pmatrix} \tag{M9}$$

$$\begin{pmatrix} (0.5063, -0.0691) & (-0.0580, \ 0.0509) \\ (-0.0580, \ 0.0509) & (0.7645, -0.4976) \end{pmatrix} \tag{M10}$$

where matrices M8, 9, and 10 are analogous to matrices M2, 3, and 4.

Matrices M8, 9, and 10, when converted to Gamma-R parameters, using (4), become

$$\begin{pmatrix} (2.5667, -1.3872) & (3.2958, -1.0764) \\ (2.2411, -0.7319) & (2.9694, -0.5303) \end{pmatrix} \tag{M11}$$

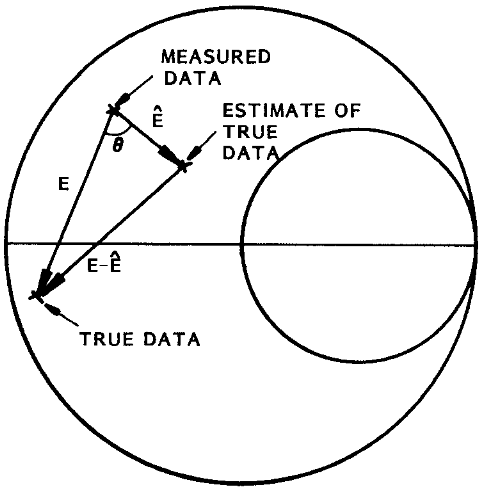

Fig. 4. The measured data is different from the true data by the measurement error E. The estimate of the error $\hat{E}$ is used to create an estimate of the true data.

$$\begin{pmatrix} (2.5669, -1.3874) & (-0.1499, \ 0.1202) \\ (-0.1019, \ 0.0818) & (-0.0464, -0.3021) \end{pmatrix} \quad \text{(M12)}$$

$$\begin{pmatrix} (2.9695, -0.5304) & (-0.1411, \ 0.0921) \\ (-0.1412, \ 0.0921) & (-0.0463, -0.3021) \end{pmatrix}. \quad \text{(M13)}$$

Combining M11, 12, and 13, we form the overall 3×3 Gamma-R matrix

$$\begin{pmatrix} (2.5666, -1.3872) & (3.2957, -1.0764) & (-0.1499, \ 0.1202) \\ (2.2411, -0.7319) & (2.9695, -0.5303) & (-0.1411, \ 0.0921) \\ (-0.1019, \ 0.0818) & (-0.1412, \ 0.0921) & (-0.0464, -0.3021) \end{pmatrix}. \quad \text{(M14)}$$

Conversion of the 3×3 Gamma-R matrix, M14, to S-parameters, using (5) yields the S-parameter matrix, M1.

All above calculations are single precision; accumulated round-off error may change the last digit.

Appendix IV
Convergence of the Iterative Algorithm

As illustrated in Fig. 4, the iterative algorithm will form an estimate $\hat{E}$ of the actual error E. The estimate of the error will be subtracted from the measured data to form a new estimate of the true data. If the new estimate of the true data is closer to the true data (i.e., if $|E - \hat{E}| < |E|$) then the algorithm will converge.

The condition for convergence can also be stated as

$$\left| \frac{\hat{E}}{E} \right| \leqslant 2 \cos \theta .$$

This condition will restrict $|E - \hat{E}|$ to within a circle of radius $|E|$ and centered on the true data.

The true error E and the estimate $\hat{E}$ for a three-port are

$$E = \frac{S_1 S_2 \Gamma}{1 - S_3 \Gamma} \qquad \hat{E} = \frac{(S_1 + e_1)(S_2 + e_2)\Gamma}{1 - (S_3 + e_3)\Gamma}$$

where S_1, S_2 are the transmission S-parameters, S_3 is the reflection S-parameter, e_1, e_2, e_3 are measurement errors, and Γ is the reflection coefficient of load.

The convergence condition becomes

$$\left| \frac{\hat{E}}{E} \right| = \left| \frac{(S_1 + e_1)(S_2 + e_2)(1 - S_3 \Gamma)}{S_1 S_2 [1 - (S_3 + e_3)\Gamma]} \right| \leqslant 2 \cos \theta .$$

Remember that θ (Fig. 4) is $\arg(\hat{E}/E)$.

We will investigate the worse case when all transmission parameters have a magnitude equal to the largest magnitude T, and all reflection coefficient magnitudes are equal to the largest magnitude R. If any S-parameter magnitude is in fact less, then the maximum errors in the measured data will be less and the algorithm will be inherently more stable.

Further, the worst case will occur at $\theta = 0$ provided that

$$|e_1| < |S_1| \quad |e_2| < |S_2| \quad |e_3 \Gamma| < |1 - S_3 \Gamma| .$$

This may be seen by drawing a vector diagram of S_1, e_1, and $S_1 + e_1$. At the worst case, $\theta = 0$ and S_1 and e_1 will be in the same direction to maximize the error. Any change in $\arg(e_1)$ will then cause $|S_1 + e_1|$ to decrease more rapidly than $2\cos(\theta)$.

Now, using the worst case and forcing Γ to be real (by shifting the reference plane), we may calculate the largest value of Γ which will guarantee that $|\hat{E}/E| < 2$ at $\theta = 0$

$$\max |e_i| = \frac{T^2 \Gamma}{1 - R\Gamma}$$

$$\max \left| \frac{\hat{E}}{E} \right| = \frac{(1 + (T - R)\Gamma)^2}{(1 - R\Gamma)^2 - T^2 \Gamma^2} \leqslant 2, \qquad \Gamma \leqslant 1/(R + 3T) .$$

If we assume a passive three-port (i.e., $T = R = 1$), then the iterative algorithm is guaranteed to converge monotonically for $|\Gamma| \leqslant 0.25$. Numerical simulation confirms this and, in fact, the iterative algorithm simulation will converge for the worst case with $|\Gamma| \leqslant 0.28$, although not monotonically. It should be emphasized that the worst case will almost never be seen in measured data, and loads with larger reflection coefficients will nearly always allow convergence, especially if only one of the S-parameters is as large as R or T.

The numerical simulation also suggests a simple test for convergence. When the algorithm failed to converge, it would alternate between two widely separated points or diverge. Thus a sufficient test for convergence may be to note when the difference between successive iterations is negligible.

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References

[1] W. Kruppa and D. F. Sodomsky, "An explicit solution for the scattering parameters of a linear two-port measured with an imperfect test set," *IEEE Trans Microwave Theory Tech.*, vol. MTT-19, pp. 122–123, Jan. 1971.

[2] S. Rehnmark, "On the calibration process of automatic network analyzer systems," *IEEE Trans Microwave Theory Tech.*, vol. MTT-22, pp. 457–458, Apr. 1974.

[3] N. R. Franzen and R. A. Speciale, "A new procedure for system calibration and error removal in automated S-parameter measurements," in *Proc. 5th Eur. Microwave Conf.*, (Hamburg, Germany), Sept. 1–4, 1975, pp. 69–73.

[4] R. A. Speciale, "Generalization of the TSD network analyzer calibration procedure, covering n-port measurements with leakage," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1100–1115, Dec. 1977.

[5] H. J. Carlin, "The scattering matrix in network theory," *IRE Trans. Circuit Theory*, vol. CT-3, pp. 88–97, June 1956.

[6] D. C. Youla, "On scattering matrices normalized to complex port numbers," *Proc. IRE*, vol. 49, p. 1221, July 1961.

[7] D. C Youla, "An extension of the concept of scattering matrix," *IEEE Trans. Circuit Theory*, vol. CT-11, pp. 310–312, June 1964.

[8] R. A. Rohrer, "The scattering matrix: Normalized to complex n-port load networks," *IEEE Trans. Circuit Theory*, vol. CT-12, pp. 223–230, June 1965.
[9] W. R. Wohlers, "Complex normalization of scattering matrices and the problem of compatible impedances," *IEEE Trans. Circuit Theory*, vol. CT-12, pp. 528–535, Dec. 1965.
[10] M. R. Wohlers, "On scattering matrices normalized to active n-ports at real frequencies," *IEEE Trans. Circuit Theory*, vol. CT-16, pp. 254–256, May 1969.
[11] D. Woods, "General renormalisation transforms for 3-port S-parameters," *Electron. Lett.*, vol. 6, pp. 823–825, Dec. 10, 1970.
[12] D. Woods, "Multiport-network analysis by matrix renormalisation employing voltage-wave S-parameters with complex normalisation," *Proc. Inst. Elec. Eng.*, vol. 124, pp. 198–204, Mar. 1977.
[13] D. Woods, "Reappraisal of computer-corrected network analyser design and calibration," *Proc. Inst. Elec. Eng.*, vol. 124, pp. 205–211, Mar. 1977.
[14] D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*. New York: Pergamon, 1967, p. 108.
[15] D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*. New York: Pergamon, 1967, p. 25.
[16] D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*. New York: Pergamon, 1967, p. 57.
[17] T. Y. Otoshi, "On scattering parameters of a reduced multiport," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 722–724, Sept. 1969.
[18] T. J. Cullen, "An elementary proof of a result used by Otoshi," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, p. 171, Mar. 1970.
[19] T. Nemoto and D. F. Wait, "Microwave circuit analysis using the equivalent generator concept," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 866–873, Oct. 1968.
[20] J. K. Hunton, "Analysis of microwave measurement techniques by means of signal flow graphs," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 206–212, Mar. 1960.
[21] W. K. Chen, "Relationship between scattering matrix and other matrix representations of linear two-port networks," *Int. J. Electron.*, vol. 38, pp. 433–441, Apr. 1975.
[22] J. G. Evans, "Linear two-port characterization independent of measuring set impedance imperfections," *Proc. IEEE*, vol. 56, pp. 754–755, Apr. 1968.
[23] J. G. Evans, "Measuring frequency characteristics of linear two-port networks automatically," *Bell Syst. Tech. J.*, vol. 48, pp. 1313–1338, May–June 1969.
[24] J. C. Tippet and R. A. Speciale, "A rigorous technique for measuring the scattering matrix of a multiport device with a 2-port network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 661–666, May 1982.

Low-Noise, Low Power Dissipation GaAs Monolithic Broad-Band Amplifiers

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Abstract —**Low-noise, low dc power dissipation GaAs monolithic amplifiers have been developed for use in VHF–UHF mobile radio systems. The developed amplifiers have two-stage construction, where gate width for the first stage is 1000 μm, and for the second stage is 500 μm. Using this circuit configuration, both noise figure and bandwidth have been improved. To maintain the uniformity for the ion-implanted active layers and to reduce gate–source resistance R_S and gate–drain resistance R_D, the "closely spaced electrode FET" was adopted. The FET enables low drain voltage operation, resulting in low dc power dissipation.**

The developed amplifier for the FET threshold voltage $V_T = -0.6$ V provides a 3-dB noise figure, less than 170-mW dc power dissipation, 9-MHz–3.9-GHz bandwidth with 16-dB gain. It can operate under a unipolar power source. When external choke inductors were introduced for the amplifier, 120-mW dc power dissipation has been achieved. It has also been demonstrated that the amplifier for $V_T = -0.6$ V, which is inferior to the amplifier for $V_T = -2.7$ V regarding gain-bandwidth product and power efficiency under the same dc power dissipation, however, has an acceptable performance for use in the mobile radio systems.

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I. Introduction

Recent advances in GaAs IC technology make it possible to develop multistage GaAs monolithic broad-band amplifiers for general purpose utilization. Expected application fields for the amplifiers are the following:

1) 1-GHz and 2-GHz band mobile radio system;
2) 1.6 = Gbit/s data rate optical communication system;
3) 3-GHz band phased array radar system;
4) intermediate frequency section in microwave communication system;
5) VHF–UHF television.

For these applications, low input and output VSWR, low-noise figure, low dc power dissipation, and high gain are required over a wide frequency range, where there are tradeoff relations among the amplifier characteristics.

Especially in the mobile radio systems, low dc power dissipation (below 150 mW) is a basic requirement for low-noise (less than 3-dB noise figure) broad-band (up to 3 GHz) amplifiers. However, dc power dissipation for the reported amplifiers was as high as 300 mW to 1600 mW [1], [3]–[6]. In addition, the noise figure was not low enough [1], [3], [4], bandwidth was not sufficiently wide [3], [5], [6], and input VSWR was not reduced [5], [9]. Accordingly, to realize GaAs monolithic broad-band amplifiers for use in mobile radio systems, a low dc power dissipation technique has to be developed, considering noise figure, bandwidth, gain, and VSWR. In addition to this, if possible, realizing a unipolar power-source operation for the amplifiers makes them very useful, from a practical point of view.

This paper describes design considerations and performances for newly developed low-noise, low power-dissipation GaAs monolithic broad-band amplifiers for use in VHF–UHF mobile radio systems. It will be shown that, by using the achieved theoretical results which have already been published [1], both noise figure and bandwidth can be improved. The developed amplifiers have two-stage construction, where gate width for the first stage is 1000 μm and that for the second stage is 500 μm. In amplifier fabrication, to improve uniformity for FET active layers and resistive layers, an ion-implantation technique was introduced. The so-called "closely spaced electrode FET" structure, which has been developed for E/D-type GaAs digital IC's [8] in the NEC Research Laboratories, is adopted so that both gate–source resistance R_s and gate–drain resistance R_D can be reduced without recessing the gate. By reducing R_S and R_D, the FET's can operate under low drain voltage with appropriate transconductance g_m, resulting in low dc power dissipation. The nonrecessed FET's maintain uniformity for the ion-implanted layers.

Also, two GaAs monolithic amplifier categories, one for unipolar power-source operation (needs only positive bias supply, $+V_D$), the other for bipolar power-source operation (needs both negative and positive bias supply, $-V_G$ and $+V_D$), are discussed comparatively. It will be demonstrated that the unipolar power-source amplifier, which is inferior to the bipolar power-source