A New Definition of Characteristic Impedance

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ABSTRACT

Existing two-dimensional definitions of characteristic impedance can result in a wide range of values for the characteristic impedance of a given transmission line in inhomogeneous media. This paper suggests a three-dimensional definition, the "TEM Equivalent" characteristic impedance, which is unique for any given transmission line geometry and is appropriate for use in circuit theory applications. Comparisons with two-dimensional results and comparisons with measurements are presented. The TEM equivalent characteristic impedance also shows a non-monotonic dispersion which is not seen in the usual two-dimensional definitions but is seen in experimentally.

INTRODUCTION

The characteristic impedance of a transmission line is usually defined in terms of field quantities along a transverse cross section, Figure 1. For homogeneous media, these definitions provide unique values which, when combined with the line's electrical length, can completely characterize the line for circuit theory applications. For inhomogeneous media, such as microstrip, the resulting characteristic impedance depends not only on the definition used (Voltage-Current, Voltage-Power, Power-Current), but also on the path of integration employed (1). The microwave circuit designer is then left to choose, by some criterion, which definition and which path of integration is "best".

There have been unpublished suggestions that the best definition of characteristic impedance depends on what is connected to each end of the transmission line. This is an undesirable situation and is, in fact, inappropriate in that the characteristic impedance is completely determined by a single mode propagating on the transmission line. The characteristics of that mode are not dependent on what structures terminate the ends of the line and thus such structures should have no influence on the characteristic impedance.

We describe a definition of characteristic impedance which requires a three-dimensional electromagnetic analysis of a transmission line and, thus, is referred to as a three-dimensional definition of characteristic impedance. No reference to cross-sectional fields is required.

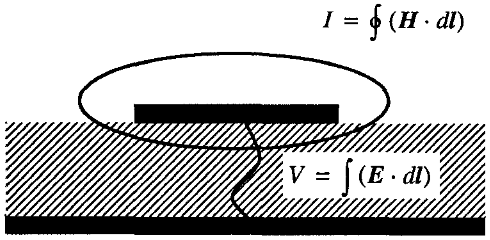

Figure 1. Classical two-dimensional definitions of characteristic impedance are functions of the cross-sectional fields. For inhomogeneous media the resulting impedances depend on the path of integration.

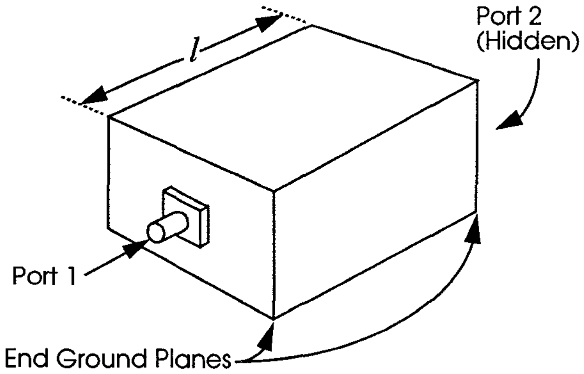

Figure 2. A length, *l*, of transmission line may contain any two-dimensional geometry between two end ground planes. Port 1 is indicated conceptually with a coaxial connector. The ports must be of circuit theory dimensions.

APPROACH

A length of transmission line (Figure 2) is here defined as any structure whose geometry has no variation along one dimension (usually the Z direction, in rectangular coordinates) for the length, *l*, of the line. The ends of the transmission line are defined by perfect ground planes covering the entire cross-section of the transmission line. The ground planes at each end must be at the same potential, i.e., a shorting "ground" must connect both ground planes and is used as the ground reference.

The ground reference need not surround the entire transmission line and may be placed at considerable distance from the line.A ground reference is required so that results may be used in a circuit theory based analysis. The ground planes at either end precisely define the length of the line and provide ground terminals for the circuit theory ports.

The ports (i.e., the "sources" in an electromagnetic analysis) must be of circuit theory dimensions, small with respect to wavelength. For example, a port voltage can be determined by integration of the tangential electric field over a single subsection in a subsectional method of moments analysis (5). With the source subsection adjacent to an end ground plane, the terminal voltage is the voltage between ground and the input to the transmission line. Integration of electric field through a substrate is not needed and is not acceptable (such a port would no longer have circuit theory dimensions).

In analogy with physical measurements, we assume that only the external circuit quantities (i.e., current and voltage at the port terminals) are available. Figure 3 shows a TEM equivalent model for the transmission line of Figure 2. Each port discontinuity is represented by a shunt capacitor. Our task is to select the TEM equivalent characteristic impedance, velocity of propagation and port discontinuity which correctly characterize the length of transmission line in terms of external circuit quantities.

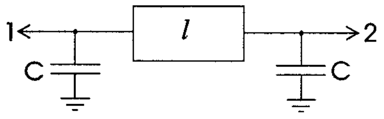

Figure 3. The equivalent circuit theory model of a length, *l*, of transmission line includes a port discontinuity at each end of an ideal TEM transmission line.

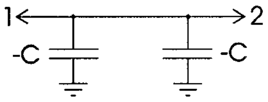

Figure 4. The resulting inverted ABCD matrix of the cascade of two port discontinuities has the above equivalent circuit. The ABCD matrix for a single port discontinuity is unique if the port discontinuities have no series impedance.

As described in (2), this characterization may be realized by evaluating lines of two different lengths (e.g., *l* and 2*l*). A necessary condition for a unique result is that the port discontinuity contains no series impedance component, only a shunt admittance (2), as illustrated in Figure 3. Thus, if the analysis allows a transverse component of current at the port, the analysis cannot be used with this technique to evaluate characteristic impedance. Note that a transverse component of current at the port also usually indicates a structure which is at variance with the previously specified definition of a length of transmission line. Note that this restriction also effectively eliminates the use of this technique in a physical measurement. Other techniques, such as in (1), must be used.

Once the above conditions are met, the evaluation of characteristic impedance proceeds as follows:

1. Using an appropriate electromagnetic analysis, evaluate the circuit theory parameters of a transmission line of length *l* and 2*l*.
2. Convert the result to ABCD-parameters.
3. Invert the ABCD-parameters of the *l* length line.
4. Pre- and post-multiply the ABCD-parameters of the 2*l* length line by the inverted ABCD-parameters of the *l* length line. The result is illustrated in Figure 4.
5. To convert the inverted double port discontinuity of Figure 4 into an inverted single port discontinuity, simple divide C (in the ABCD-parameters) by 2. Note that if there is any series component in the port discontinuity (i.e., A or B ≠ 1.0 or B ≠ 0.0), this step has no unique solution.
6. Pre- and post-multiply the *l* length ABCD-parameters by the inverted single discontinuity ABCD matrix.

The ABCD parameters of the *l* Length line with port discontinuities removed is the result of the last step above. This technique forms the basis of an electromagnetic de-embedding technique which was developed in 1987 and has been in use in commercial software (5) since then. A similar technique (4) also appears to be under development. The technique is easily extended to N-coupled lines by viewing A, B, C, and D as NxN sub-matrices of a 2Nx2N ABCD matrix.

To determine the TEM Equivalent characteristic impedance of the line we use the ABCD-parameters of an ideal TEM transmission line:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\beta l) & jZ_0 \sin(\beta l) \\ j\dfrac{\sin(\beta l)}{Z_0} & \cos(\beta l) \end{bmatrix}$$

where βl is the electrical length of the line (radians) and

Z_0 is the characteristic impedance of the line.

With the ABCD-parameters of the de-embedded *l* length transmission line, the above equation can be used to evaluate Z_0, $\cos(\beta l)$, and $\sin(\beta l)$, provided *l* is not an odd multiple of a half wavelength. The effect of dispersion is determined by evaluating these parameters as a function of frequency.

The Z_0 which results from setting the TEM ABCD matrix equal to the de-embedded inhomogeneous transmission line ABCD matrix is what we call the "TEM Equivalent Impedance" in that either this impedance (and βl) or the 3-D electromagnetic analysis can be used to determine the circuit theory port quantities.

THE "BEST" IMPEDANCE DEFINITION

When using previous definitions of characteristic impedance, the microwave designer is forced to decide which is "best". An explicit criterion for judging "best" has never been published, so we suggest the following:

> "The best characteristic impedance is that which when applied to the equations for an ideal TEM transmission line predicts circuit theory parameters which are as close as possible to their actual values."

What we have done is turned that objective into the definition of characteristic impedance. Because evaluation of the characteristic impedance according to this definition requires a three-dimensional (in terms of fields) electromagnetic analysis, it can be viewed as a three-dimensional definition of characteristic impedance. This definition is fully valid and unique for all inhomogeneous transmission line media. Further, for homogeneous media, it is equivalent to the two-dimensional impedance definitions.

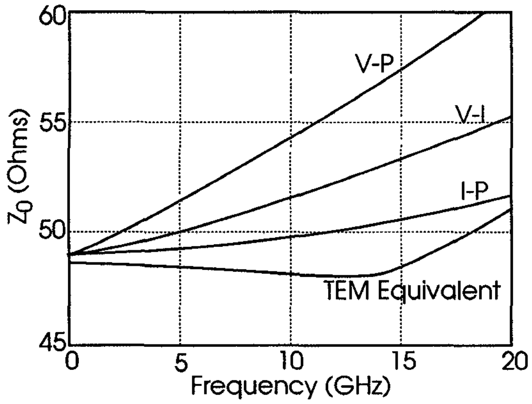

Figure 5. Plot of TEM equivalent characteristic impedance for a 0.635 mm wide line on 0.635 mm thick ε_{rel}= 9.7 substrate. Also plotted are three of the usual 2-D impedances (from (3)): Voltage-Power, Voltage-Current, and Current-Power.

RESULTS

The electromagnetic analysis *em* (5) was used to determine the TEM Equivalent characteristic impedance of several transmission lines. *Em* is commercially available and has had the above described de-embedding algorithm implemented for nearly four years. It has been extensively validated in the industry.

Figure 5 shows the resulting non-monotonic behavior of the TEM equivalent characteristic impedance. This behavior is not seen in the two-dimensional definitions of impedance also plotted.

Figures 6 and 7 are the key figures in this paper. They show calculations compared to experimental results (1). While the TEM Equivalent impedance is typically a fraction of an Ohm higher than the measured data, the two-dimensional impedances (scaling frequency and size by 10) from (3), plotted in Figure 5 are up to 10 Ohms higher than the measurements in Figure 7. Both TEM Equivalent and measurements show same the non-monotonic behavior.

The reason for the non-monotonic behavior is not known. A conceptual rationalization is that as frequency increases, electric fields tend to concentrate in the substrate, increasing the capacitance and reducing the characteristic impedance. At a certain frequency, current concentrating on the edges becomes dominant, increasing the series inductance and increasing the characteristic impedance. In both cases the velocity of propagation decreases monotonically.

CONCLUSION

We have shown how a new de-embedding technique, appropriate for electromagnetic analyses, results in a new three-dimensional definition of characteristic impedance, the TEM Equivalent characteristic impedance. This new definition is appropriate and unique for all inhomogeneous, as well as homogeneous, media. Further, the resulting characteristic impedance, for a specific microstrip geometry is shown to have a non-monotonic dispersion in characteristic impedance. This same dispersion is seen in measurements and is not seen in any of the classical, two-dimensional definitions of impedance.

REFERENCES

[1] W. J. Getsinger, "Measurement and modeling of the apparent characteristic impedance of microstrip," IEEE Trans. Microwave Theory Tech., vol. MTT-31, pp. 624-632, Aug. 1983.

[2] J. C. Rautio, "A de-embedding algorithm for electromagnetics," accepted for publication in International Journal of Microwave and Millimeter-Wave Computer Aided Engineering, Vol. 1, No. 3.

[3] R. H. Jansen and N. H. L. Koster, "New aspects concerning the definition of microstrip characteristic impedance as a function of frequency," IEEE MTT-S International Microwave Symposium Digest, 1982, pp. 305-307.

[4] A. Hill, private communication, May 1990, Syracuse, NY.

[5] **The Em User's Manual**, Sonnet Software, Inc., May1990.

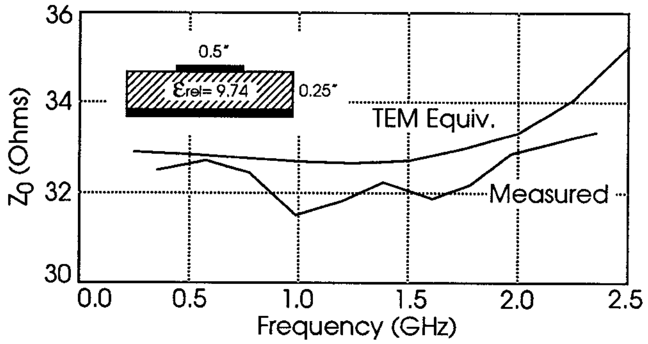

Figure 6. Average difference between measured data (1) and calculated data is about 0.5 Ohms. Both show non-monotonic behavior.

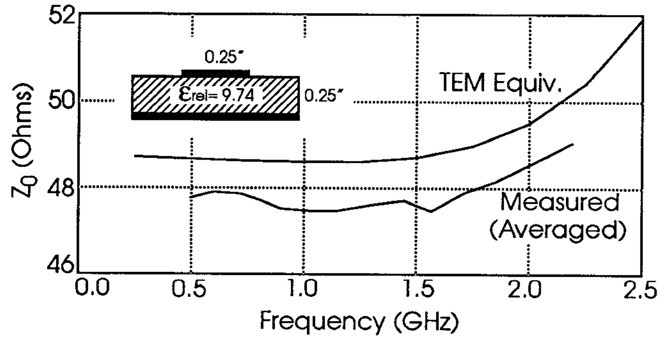

Figure 7. Measured data (1) is about 0.8 Ohms lower than the calculated TEM Equivalent Impedance.Differences are within the scatter of the original unaveraged data. Both show non-monotonic behavior.